现代数学四大猜想的直接证明

刘鸿高著

★ ★ ★ ★ ★

THE DIRECT PROOF OF FOUR GREAT CONJECTURES IN MODERN MATHEMATICS

BY HONGGAO LIU

费尔马猜想、四色猜想和哥德巴赫猜想被称为现代数学的三大难题，而冰雹猜想则是二十世纪世界著名的数学难题。它们当中，哥德巴赫猜想和冰雹猜想还没有其他人的任何证明；而费尔马猜想和四色猜想虽然有其他人的间接证明，但都还没有得到公认。这本书就是现代数学四大猜想的五个直接证明。

Fermat Conjecture, Four–Colour Conjecture and Goldbach Conjecture are known as three most difficult great problems in modern mathematics. And the Hail Conjecture is a famous mathematical problem in 20th century. Among them, Goldbach Conjecture and Hail Conjecture have not any else s proof ; Although Fermat Conjecture and Four colour Conjecture have elses indirect proof, but they not have get general acknowledgement all still. This book is five direct proofs of four great conjectures in modern mathematics.

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现代数学四大猜想的直接证明

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THE DIRECT PROOF OF FOUR GREAT CONJECTURES IN MODERN MATHEMATICS

BY HONGGAO LIU

科華圖書出版公司

“人的本性就是求知。”

——亚里士多德

“在这里，皇帝没有特权。”

——欧几里得

“数学是上帝的语言。”

——伽利略

数学的每一个正确命题和论证都是一个真理。

最高的数学就是最高的诗。

99% 的心血 +1% 新方法的灵感 = 成功。

——作者

“Human natural instincts are seek knowledge.”

——Aristotle

“Emperor not has privilege in this.”

——Euclid

“Mathematics is God″ s language.”

——Galileo

Every one correct proposition and demonstration of mathematics is a truth.

The most advanced mathematics is the most advanced poetry.

99% blood of heart + 1% inspiration of new method = success.

——The author

作者照片
The author photograph

作者简介

刘鸿高：男性， 1936 年 12 月出生，广东省连平县人， 1962 年大学五年制本科毕业，高级工程师， 1993 年 9 月以前在湖南省工作，曾任县处级总工程师，1993 年 9 月调到广东省河源市建设委员会任职， 1999 年 7 月退休后专门从事科学研究与写作。主要论著有：

（ 1 ）《地下水动力学涌水量计算公式的研究》（ 1977 年）；

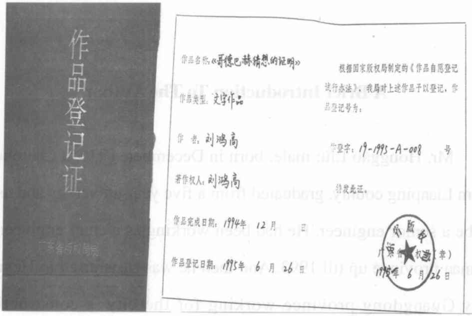
（ 2 ）《地下水水力学计算》（ 1981 年）；

（ 3 ）《现代数学四大猜想的直接证明》（ 2005 年）。

A Brief Introduction To The Author

Mr. Honggao Liu: male, born in December, 1936, a Cantonese from Lianping county, graduated from a five year university and used to be a senior engineer. He had been working as a chief engineer in Hunan province up till 1993. And then he was transferred to Heyuan city Guangdong province working for the city′ s construction commission. He retired from his working post in 1999 and has been engaging in scientific research and writing since then. His main scientific research works are following:

1. "The Research Of Calculating Formula For The Dynamics Gushing Amount On Underground Water " (1977);
2. "A Hydrodynamic Calculation Of Underground Water" (1981);
3. "The Direct Proof Of Four Great Conjectures In Modern mathematics" (2005).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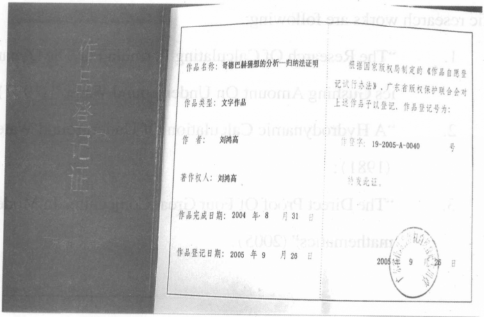
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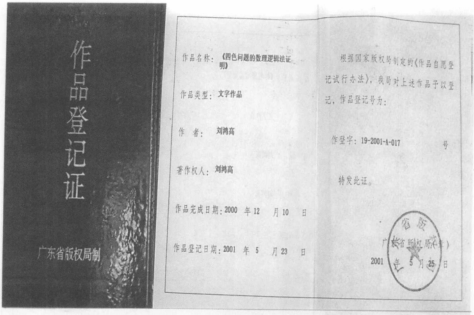
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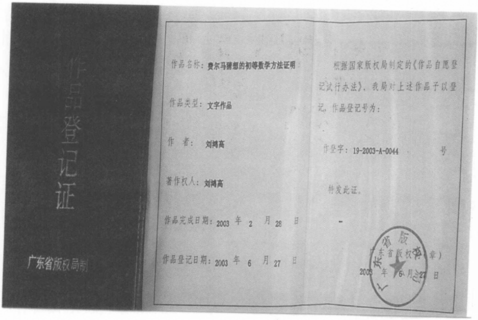
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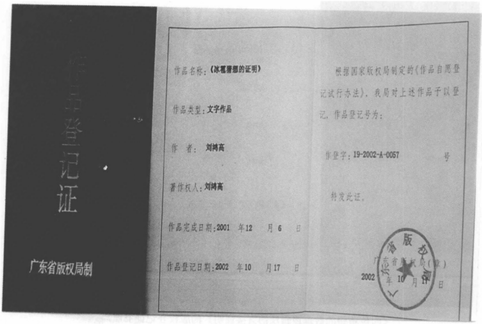

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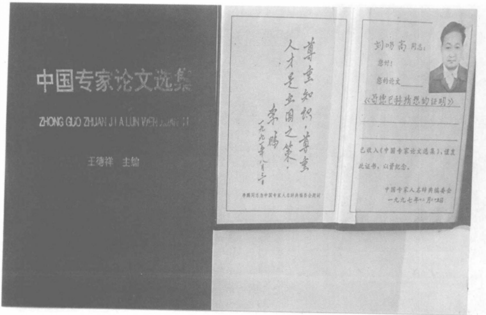

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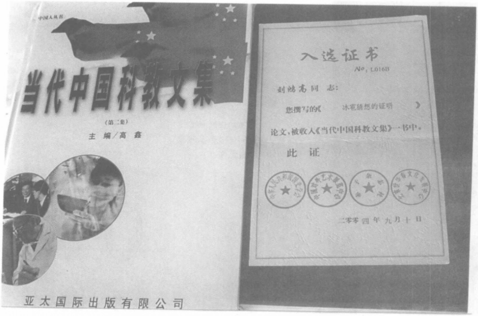

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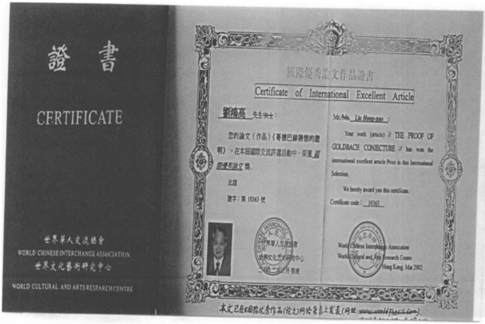

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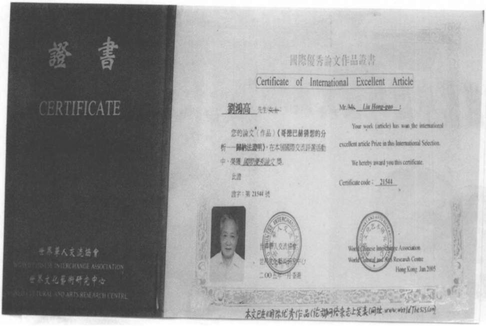

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前言：我是“数学疯子”和“傻瓜”

费尔马猜想、四色猜想和哥德巴赫猜想被称为现代数学的三大难题，而冰雹猜想则是二十世纪世界著名的数学难题。它们当中，哥德巴赫猜想和冰雹猜想还没有其他人的任何证明；而费尔马猜想和四色猜想虽然有其他人的间接证明，但都还没有得到公认。这本书就是现代数学四大猜想的五个直接证明。其中哥德巴赫猜想不仅能够被直接证明，而且还有两种直接证明。我单独或者综合运用初等数学、初等数理逻辑、初等数论的方法，直接证明了现代数学的四大猜想。我的证明是严密的，在文字上是简单明了的。凡有高中知识的人都能够读懂五篇论文。它们是真理还是谬论，我估计我国有一亿多人、而世界有十亿以上的人都有同等资格可以做出正确判断。这正如欧几里得曾经说的：“在这里，皇帝没有特权”。

我在 1995 年 2 月首次发表《哥德巴赫猜想的证明》（现在已改成《哥德巴赫猜想的同余分类篩法证明》）时，一个记者在报道中转弯抹角地说我是“数学疯子”。他说只有“数学疯子”才胆敢写那样的文章。现在我把这篇论文与以前写的一篇论文和以后写的三篇论文集结出版，严正宣告：现代数学的四大猜想都得到了一般的直接证明。由于这个原因，按照那个记者的逻辑，我就是双料又双料的“数学疯子”了。

我于 1999 年 7 月退休后，拒绝了有优厚待遇的工作聘请而专门

从事科学研究，有人说我是“傻瓜”。而且我研究的课题又是纯理论的现代数学难题，不仅影响生活和休息，不实用，没有经济效益，还要花钱，就是取得了成果也很难得到学术界的公认，那我就是“最大的傻瓜”了。

我的五篇论文都已经在我国分别进行了版权登记，而且都已经分别单独地出版了一至四次。这本书之所以同时用中文和英文出版，正因为：

对于现代数学的四大猜想，

我有充分必要的理由坚信：

它们得到了一般直接证明，

世界人们一定会普遍承认。

作者二OO五年十月

Foreword: I Am A "Mathematics Madman" And "Fool"

Fermat Conjecture, Four–Colour Conjecture and Goldbach Conjecture are known as three most difficult great problems in modern mathematics. And the Hail Conjecture is a famous mathematical problem in 20^{th} century. Among them, Goldbach Conjecture and Hail Conjecture have not any else' s proof ; Although Fermat Conjecture and Four—colour Conjecture have else' s indirect proof, but they not have get general acknowledgement all still. This book is five direct proofs of four great conjectures in modern mathematics. Among them, Goldbach Conjecture not only can be direct proved, but it has two direct proofs still. I direct proved these four great conjectures in modern mathematics by means of elementary mathematics, elementary mathematical logic, and elementary number theory singly or comprehensively. My proving is rigorous and its expression is simple and easy to understand. Those who have high school schooling can read and understand these five treatises. They are correct or wrong, I guess, can be judged by a hundred million Chinese and one billion people in the world. This as Euclid had said: "emperor not has previlege in this".

In February 1995 when I published "The Proof Of Goldbach Conjecture"(now it has been changed as "The Proof Of Goldbach Conjecture By Sieve Method Of Congruence Classification") for the first time, a reporter wrote in his report in a roundabout way saying that I am a "mathematics madman". He said that only by mathematics madman dared to write such paper. Now I publish this treatise together with preceding one treatise and later three treatises as a collection to declare severely that four great conjectures in modern mathematics have been proved generally directly. For this reason, according to that reporter' s logic, I am a double and double "mathematics madman".

Since I retired from my working post in July 1999, I have been engaging in scientific research intently instead of being invited to further my engineering work with high pay. Some people said that I am a "fool". They thought my research is purely theoretical on modern mathematical problems, which takes me a lot of time and energy affecting my life and rest, and is not practical without any economic result. They said that even though my book is published, it is also very hard to be widely recognized by the academic circle, therefore I am really "the biggest fool".

My five treatises copyright was registered in China respectively. And these treatises all were published from 1 to 4 times separately. The reason why I publish my book both in Chinese and English is just that:

As to the proof of four great conjectures in modern mathematics,
I have every necessary reason to believe resolutely:
They have been proved generally directly,
World people are definite able to universal recognition.

The author
October,2005

哥德巴赫猜想的同余分类筛法证明※

提要：本文作者首先创造了一种新的同余分类筛法，然后证明了五条新的定理，从而一般地直接证明了哥德巴赫猜想命题（1）：每个大于 2 的偶数都是两个素数之和；进而一般地直接证明了哥德巴赫猜想命题（2）：每个大于 5 的奇数都是三个素数之和。哥德巴赫猜想完全得到了一般的直接证明。

主题词：哥德巴赫猜想 同余分类筛法 五条新定理 一般直接证明

※这篇论文的第一稿首先发表于 1995 年 2 月 9 日的《河源报》，1995 年 6 月 26 日在我国进行了版权登记，登记证号码为作登字：19—1995—A—008 号；第二稿已于 1997 年 12 月在《中国专家论文选集》中出版；第三稿已于 1999 年 11 月 26 日上因特网，网址为 http://www.hy.sti.gd.cn/kyl.html。它获得了国际优秀论文奖，已于 2002 年 3 月在网络杂志《国际优秀论文》上发表，网址为：www.InteroutstandingPaper.com.

一、序言

哥德巴赫猜想是数论的著名问题之一，由德国数学家哥德巴赫（Christian Goldbach）1742 年 6 月 7 日在给欧勒的信中提出。当时由于 1 也是素数，他提出两个命题：（1）每个偶数都是两个素数之和；（2）每个大于 1 的奇数都是三个素数之和。后来国际数学家大会决定 1 不是素数，因而哥德巴赫猜想的两个命题应该改写成：（1） 每个大于 2 的偶数都是两个素数之和① ；（2） 每个大于 5 的奇数都是三个素数之和[1]。两百多年来，我国和外国的许多数学家为了证明这个猜想而奋斗不息，但还没有对该猜想做出一般的证明。这个猜想的一般证明确实十分复杂，本文作者从 1978 年春开始对它进行研究，用了整整 17 年时间才把它解决，而且又用了 5 年时间才定稿。作者首先创造了一种新的同余分类筛法，然后证明了五条新的定理，从而一般地证明了哥德巴赫猜想完全成立。

二、命题（1）的证明

命题（1）：每个大于 2 的偶数都是两个素数之和。

为了证明命题（1），需从整数开始进行系统的研究。整数以 2 为模，分为偶数和奇数两类。偶数除 2 为素数以外，其它都是合数。奇数包含了大于 2 的所有素数和许多合数。

4 在大于 2 的偶数中是一个特例。它由偶素数 2 构成 2P(P 为素数)型偶数，其“1+1”解为： 4＝2＋2。

对于大于 4 的偶数，素数 2 都不能构成“1+1”解。因此，在研究大于 4 的偶数的“1+1”解时，根本不用考虑 2 素数，而只能在大于 2 的奇素数中进行。

在大于 2 的任何区间的整数中，也就是说在大于 4 的偶数的中数以内[2]都有奇素数，而且奇素数的数量无限[2]。这就为证明大于 4 的偶数都有一般的“1+1”解提供了前提条件。

为了证明大于 4 的偶数 2N 都有一般的“1+1”解，需以一定的素数为模，对一定类型的数进行同余分类，然后把对一定素数模 0 同余的同余类筛选掉。这种以同余分类为基础进行筛选的方法，叫做同余分类筛法。运用同余分类筛法，根据下列五条定理，便可证明每个大于 4 的偶数都有一般的“1+1”解。

整数数列以 6 为模进行同余分类，大于 3 的素数只能出现在 $6m\pm1(m>0)$ 两个同余类中，也就是说，都可以表述为 $6a-5(a>1)$ 或 $6b-7(b>1)$ 类型的数，而对于上述两个类型的数有定理 1 。

定理 1 、设 P_c 为大于 3 的素数，在 6a-5 或 6b-7 类型数的数列中，

任意取P_c个以内的连续数，则其中最多只有一个数能够使得$6a \equiv 5 (\bmod P_c)$或$6b \equiv 7(\bmod P_c)$成立，其它各个数都不能使得$6a \equiv 5(\bmod P_c)$或$6b \equiv 7(\bmod P_c)$成立。

证明：6a-5 和 6b-7 类型数的数列都是等差为 6 的等差级数。在 6a-5 和 6b-7 类型数的数列中，因为$2|6a$，$2|6b$，而 2 不能整除 5，2 不能整除 7，所以 2 不能整除 6a-5，2 不能整除 6b-7；同理 3 不能整除 6a-5，3 不能整除 6b-7。因此，在判断 6a-5 或 6b-7 类型的数是否为素数时，模数不必考虑素数 2 和 3，而只需考虑大于 3 的素数P_c。

设连续的等差为 6 的P_c个$6a-5(a>1)$类型的等差级数为：$6(a+1)-5$，$6(a+2)-5$，$6(a+3)-5$……$6(a+P_c-2)-5$，$6(a+P_c-1)-5$，$6(a+P_c)-5$。在该等差级数的数列中，若某个数$6(a+m)-5$（$0<m\leqslant P_c$）除以大于 3 的素数P_c为整数，即$6(a+m)-5=nP_c(n>0)$；那么，其它P_c-1个数为$6(a+m)-5\pm 6e(0<e<P_c$，e 为整数)，它们除以P_c为$[6(a+m)-5]/P_c\pm 6e/P_c$，由于$P_c|6(a+m)-5$，而 6e 不含P_c因子，P_c不能整除 6e，所以P_c不能整除$6(a+m)-5\pm 6e$，各个数都总有一定的余数$r(0<r<P_c$，r 为整数)。该数列由于是等差级数，每个数的余数都不可能相等。余数的变化值为$1\sim P_c-1$，而有余数的数也是P_c-1个，所以各个数的余数必然分别为 1，2，3……P_c-3，P_c-2，P_c-1中的某一个数。这些余数的分布次序随着起始数的不同而不同，但都有一定的分布规律。

上述连续的P_c个等差为 6 的等差级数的起始数可以任意确定，如

$6(a+P_c-2)-5$，$6(a+P_c-1)-5$，$6(a+P_c)-5$，$6(a+P_c+1)-5\cdots\cdots6(a+2P_c-4)-5$，$6(a+2P_c-3)-5$ 等。

同理可以证明在连续的等差为 6 的 P_c 个 $6b-7$（$b>1$）类型的等差级数中，只有某一个数 $6(b+m)-7$（$0<m\leq P_c$）能被 P_c 整除，使得 $6(b+m)-7=nP_c$（$n>0$），而其它 P_c-1 个数除以 P_c 都不可能为整数，总有一定的余数 r。在 P_c-1 个等差级数中，各个数的余数也必然分别为 $1,2,3\cdots\cdots P_c-3$，P_c-2，P_c-1 中的某一个数。这些余数的分布也总有一定的规律。连续的 P_c 个等差为 6 的等差级数 $6b-7$ 的起始数也可以任意确定。

既然在 $6a-5$ 或 $6b-7$ 类型等差级数的数列中，有 P_c 个连续数时，只有一个数能够使得 $6a\equiv 5(\mathrm{mod}P_c)$ 或 $6b\equiv 7(\mathrm{mod}P_c)$ 成立，其它 P_c-1 个数都不能使得 $6a\equiv 5(\mathrm{mod}P_c)$ 或 $6b\equiv 7(\mathrm{mod}P_c)$ 成立，那么，在 $6a-5$ 或 $6b-7$ 类型数的数列中，只有 $2\sim P_c-1$ 个连续数时，显然就最多只有一个数能够使得 $6a\equiv 5(\mathrm{mod}P_c)$ 或 $6b\equiv 7(\mathrm{mod}P_c)$ 成立，其它各个数都不能使得

$6a\equiv 5(\mathrm{mod}P_c)$ 或 $6b\equiv 7(\mathrm{mod}P_c)$ 成立。定理 1 证明完毕。

定理 2、设 P_A 为大于 5 的一定素数，则在 $P_A\sim 2P_A$ 之间至少有两个素数。

分析：为了便于证明定理 2，必须先行了解在 $P_A\sim 2P_A$ 之间的 $6a-5$ 类型数和 $6b-7$ 类型数的连续分布情况，以及 a 与 $[2(6a-5)]^{1/2}$ 和 b 与 $[2(6b-7)]^{1/2}$ 的关系。

1、在 $P_A\sim 2P_A$ 之间的 $6a-5$ 类型数和 $6b-7$ 类型数的连续分布情况：

(1)当 P_A 为 6a-5 类型的素数时：

A、设 X 为大于 0 的整数，在 P_A～$2P_A$ 之间，6(a+x)-5 类型的数（不一定是素数）有：$6a-5<6(a+x)-5<2(6a-5)$。

由 $6a-5<6(a+x)-5$ 可知

$$6a-5<6a+6x-5,$$

$$6x>0,$$

即 $x>0$；

由 $6(a+x)-5<2(6a-5)$ 可知

$$6a+6x-5<12a-10,$$

$$6x<6a-5,$$

$$x<a-5/6,$$

只考虑整数， $x<a$ 。

综上所述，$0<x<a$ 。因此，在 P_A～$2P_A$ 之间有 a-1 个 6(a+x)-5 类型的连续数。

B、在 P_A～$2P_A$ 之间，6b-7 类型的数（不一定是素数）有：

$$6a-5<6b-7<2(6a-5)。$$

由 $6a-5<6b-7$ 可知

$$6b>6a+2,$$

$$b>a+1/3,$$

只考虑整数，即 $b>a$ ；

由 $6b-7<2(6a-5)$ 可知

$$6b-7<12a-10,$$

$$b<2a-1/2,$$

只考虑整数，即 $b<2a$ 。

综上所述：$a<b<2a$ 。因此，在 $P_A\sim 2P_A$ 之间也有 $a-1$ 个 $6b-7$ 类型的连续数。

(2)当 P_A 为 $6b-7$ 类型的素数时：

A、在 $P_A\sim 2P_A$ 之间，$6a-5$ 类型的数有：

$$6b-7<6a-5<2(6b-7)\text{。}$$

由 $6b-7<6a-5$ 可知

$$6a>6b-2,$$

$$a>b-1/3,$$

只考虑整数，$a>b-1$；

由 $6a-5<2(6b-7)$ 可知

$$6a-5<12b-14,$$

$$6a<12b-9,$$

$$a<2b-3/2,$$

只考虑整数，$a<2b-1$ 。

综上所述，$b-1<a<2b-1$ 。因此，在 $P_A\sim 2P_A$ 之间有 $a=b$，$b+1$，……，

2b-3，2b-2 等 b-1 个 6a-5 类型的连续数。

B、设 y 为大于 0 的整数，在 P_A～$2P_A$之间，6(b+y)-7 类型的数有：

$$6b-7<6(b+y)-7<2(6b-7)。$$

由 $6b-7<6(b+y)-7$ 可知

$$6b-7<6b+6y-7,$$

$$6y>0,$$

即 $$y>0;$$

由 $6(b+y)-7<2(6b-7)$ 可知

$$6b+6y-7<12b-14,$$

$$6y<6b-7,$$

$$y<b-7/6,$$

只考虑整数，即 $y<b-1$。

综上所述：$0<y<b-1$ 。因此，在 P_A～$2P_A$之间有 b-2 个 6(b+y)-7 类型的连续数。

总之，当 P_A为 6a-5 类型的素数时，在 P_A～$2P_A$之间有 a-1 个 6a-5 类型的连续数和 a-1 个 6b-7 类型的连续数；当 P_A为 6b-7 类型的素数时，在 P_A～$2P_A$之间有 b-1 个 6a-5 类型的连续数和 b-2 个 6b-7 类型的连续数。不管 P_A为 6a-5 类型的素数，还是 6b-7 类型的素数，每当 a 或 b 增大 1，则在 P_A～$2P_A$之间增加 1 个 6a-5 类型的数和 1 个 6b-7 类型的数。

2、a 与$[2(6a-5)]^{1/2}$及 b 与$[2(6b-7)]^{1/2}$的关系：

(1)a 与$[2(6a-5)]^{1/2}$的关系：随着 a 的增大，$[2(6a-5)]^{1/2}$也逐渐增大，但增加得很缓慢，所以当 a 增大到一定值时，便有：

$$a > [2(6a-5)]^{1/2},$$

$$a^2 > 12a-10,$$

$$a^2-12a+10>0 。$$

解上述不等式得：

$$a>11.1,$$

只考虑整数，即 $a>11$ 时，上述不等式成立。

(2)b 与$[2(6b-7)]^{1/2}$的关系：运用上述证明 $a>11$ 时 $a>[2(6a-5)]^{1/2}$成立的方法，同样可以由 $b>[2(6b-7)]^{1/2}$解得 $b>10$ 时该不等式成立。

以上计算表明：当 $a>11$ 时，$a>[2(6a-5)]^{1/2}$；当 $b>10$ 时，$b>[2(6b-7)]^{1/2}$。为了运用方便，不管 P_A为 6a-5 类型的素数或 6b-7 类型的素数，都可以以较大的数为准，统一规定为：当 $a>11$ 时，$a>(2P_A)^{1/2}$；当 $b>11$ 时，$b>(2P_A)^{1/2}$。因此，当 $a>11$ 或 $b>11$ 时，就可以运用 a 或 b 以内大于 3 的素数 P_j代替 $(2P_A)^{1/2}$以内大于 3 的素数，对 P_A～$2P_A$之间的 6a-5 类型的数和 6b-7 类型的数是否为素数进行判定。

证明：运用 $a>11$ 和 $b>11$，便把 P_A分成 7～61 的素数和大于 61 的素数两部分，定理 2 的证明也可分两步进行。

1、证明在 7～61 素数的数列中，任意选定一个素数 P_A，在 P_A～$2P_A$ 之间至少有两个素数 P_{A+1} 和 P_{A+2}.

在 7～61 之间，任意选定一个素数 P_A，从极有限的素数表就可以直观看出，甚至许多有数论基本知识的人在记忆中就明确知道，在 P_A～$2P_A$ 之间至少有两个素数 P_{A+1} 和 P_{A+2}，而无需去做数理逻辑的证明。实际上只是当 P_A=7 时，在 7～14 之间才只有两个素数 11 和 13；而当 P_A=11～61 时，在 P_A～$2P_A$ 之间的素数由 3 个迅速增加到 12 个。

2、证明 P_A>61 的素数，在 P_A～$2P_A$ 之间至少有两个素数 P_{A+1} 和 P_{A+2}。

P_A>61，即 a>11，b>11。在大于 61 的素数数列中，最小的 6a-5 类型的素数是 67，而最小的 6b-7 类型的素数是 71。运用同余分类筛法和归纳法有：

(1)当 P_A 为 6a-5 类型的素数时：

A、当 P_A=67，a=12，根据以上所述，在 P_A～$2P_A$ 之间，有 a-1=11 个 6a-5 类型的连续数和 a-1=11 个 6b-7 类型的连续数。a 以内包含有 5、7、11 三个大于 3 的素数。

根据定理 1，以素数 11 为模，对 11 个 6a-5 类型的连续数进行同余分类，其中只有 1 个 6a-5 类型的数能够使得 6a≡5(mod11)成立，筛选掉，其它 10 个 6a-5 类型的数都不能使得 6a≡5(mod11)成立；

其次，在以上数列中，取 7 个 6a-5 类型的连续数[可以避开、也可

以包含前面已经筛选掉的那个6a≡5(mod11)的6a-5类型的数]，以7为模，进行同余分类，也只有1个6a-5类型的数能够使得6a≡5(mod7)成立，筛选掉，纵使包括前面那个已经筛选掉的6a-5类型的数，至少还有5个6a-5类型的数都不能使得6a≡5(mod7)成立；

最后，在上述数列中，取5个6a-5类型的连续数[可以避开、也可以包括前面已筛选掉的那两个6a≡5(mod11)和6a≡5(mod7)的6a-5类型的数]，以5为模，进行同余分类，纵使同余的6a-5类型的数不重复，也只有1个6a-5类型的数能够使得6a≡5(mod5)成立，筛选掉，就是除开前面已经筛选掉的两个6a-5类型的数，至少还有两个6a-5类型的数不能使得6a≡5(mod5)成立，成为素数。

同理可证明：在67～134之间11个6b-7类型的连续数中，至少有两个6b-7类型的数都不能使得6b≡7(mod11，7，5)成立，成为素数。

综上所述，当P_A=67时，在P_A～$2P_A$之间，至少有两个6a-5类型的素数和两个6b-7类型的素数，即至少共有4个素数（实际多达13个），定理2成立。

B、当P_A>67时，a>12，在P_A～$2P_A$之间有a-1个6a-5类型的连续数和a-1个6b-7类型的连续数。

假设当a=u，u为大于12的整数，在6u-5～2(6u-5)之间有u-1个6a-5类型的连续数和u-1个6b-7类型的连续数，u以内大于3的素数为P_j，设P_u为P_j的最大值，P_u可能的最大值是P_u=u。

首先以 P_u 为模，对 u-1 个 6a-5 类型的连续数进行同余分类，根据定理 1，最多只有 1 个同余类能够使得 $6a\equiv5(\mathrm{mod}P_u)$ 成立，筛选掉，其它 u-2 个同余类都不能使得 $6a\equiv5(\mathrm{mod}P_u)$ 成立；然后以 P_{u-1} 为模，对那些不能使得 $6a\equiv5(\mathrm{mod}P_u)$ 成立的 6a-5 类型的数进行同余分类，最多也只有 1 个同余类能够使得 $6a\equiv5(\mathrm{mod}P_{u-1})$ 成立，筛选掉，其它同余类都不能使得 $6a\equiv5(\mathrm{mod}P_{u-1})$ 成立；如此继续分别依次用 P_{u-2} 、P_{u-3}……P_3、P_2 为模，对以前一个模不同余的 6a-5 类型的数进行同余分类，纵使每次都有 1 个同余类能够使得 $6a\equiv5(\mathrm{mod}P_j)$ 成立，逐步筛选掉，最后至少有 1 个同余类不能使得 $6a\equiv5(\mathrm{mod}P_j)$ 成立，即至少有 1 个 6a-5 类型的数为素数。

而且，以 P_u 为模，对 u-1 个 6b-7 类型的连续数进行同余分类，也最多只有 1 个同余类能够使得 $6b\equiv7(\mathrm{mod}P_u)$ 成立，筛选掉，其它 u-2 个同余类都不能使得 $6b\equiv7(\mathrm{mod}P_u)$ 成立；然后以 P_{u-1} 为模，对那些不能使得 $6b\equiv7(\mathrm{mod}P_u)$ 成立的 6b-7 类型的数进行同余分类，最多也只有 1 个同余类能够使得 $6b\equiv7(\mathrm{mod}P_{u-1})$ 成立，筛选掉，其它同余类都不能使得 $6b\equiv7(\mathrm{mod}P_{u-1})$ 成立；如此继续分别依次用 P_{u-2}、P_{u-3}……P_3、P_2 为模，对以前一个模不同余的 6b-7 类型的数进行同余分类，也纵使每次都有 1 个同余类能够使得 $6b\equiv7(\mathrm{mod}P_j)$ 成立，逐步筛选掉，最后至少有一个同余类不能使得 $6b\equiv7(\mathrm{mod}P_j)$ 成立，即至少有 1 个 6b-7 类型的数为素数。

当 a=u+1 时，在 6(u+1)-5～2[6(u+1)-5]之间，有 u 个 6a-5 类型的

连续数和 u 个 6b-7 类型的连续数，u+1 以内大于 3 的素数 P_j 可能与 a=u 时一样多，最多也只能比 a=u 时增加 1 个，纵使为后者，设最大的 P_j 为 P_{u+1}，P_{u+1} 可能的最大值为 u+1 。

以 P_{u+1} 为模，对 u 个 6a-5 类型的连续数进行同余分类，最多只有 1 个同余类能够使得 6a≡5(modP_{u+1})成立，筛选掉，其它 u-1 个同余类都不能使得 6a≡5(modP_{u+1})成立；然后完全可以与假设前提一样，继续分别依次以 P_u、、P_{u-1}…… P_3、、P_2 为模，对以前一个模不同余的 6a-5 类型的数进行同余分类，纵使每次都有 1 个同余类能够使得 6a≡5(modP_j)成立，逐步筛选掉，最后至少有 1 个同余类不能使得 6a≡5(modP_j)成立，即至少有 1 个 6a-5 类型的数为素数。

同样，以 P_{u+1} 为模，对 u 个 6b-7 类型的连续数进行同余分类，最多只有 1 个同余类能够使得 6b≡7(modP_{u+1})成立，筛选掉，其它 u-1 个同余类都不能使得 6b≡7(modP_{u+1})成立；然后也完全可以与假设前提一样，继续分别依次用 P_u、、P_{u-1}……P_3、、P_2 为模，对以前一个模不同余的 6b-7 类型的数进行同余分类，也纵使每次都有 1 个同余类能够使得 6b≡7(modP_j)成立，逐步筛选掉，最后至少有 1 个同余类不能使得 6b≡7(modP_j)成立，即至少有 1 个 6b-7 类型的数为素数。

综上所述，当 P_A 为大于 67 的 6a-5 类型的素数时，在 P_A～$2P_A$ 之间至少有 1 个 6a-5 类型的素数和 1 个 6b-7 类型的素数，即一共至少有两个素数。

(2)当P_A为6b-7类型的素数时：

A、当P_A=71，b=13，在P_A～$2P_A$之间，有b-1=12个6a-5类型的连续数和b-2=11个6b-7类型的连续数。b以内包含大于3的素数P_j有5、7、11、13四个。

首先，以13为模，对12个6a-5类型的连续数进行同余分类，最多只有1个同余类能够使得6a≡5(mod13)成立，筛选掉，其它11个同余类都不能使得6a≡5(mod13)成立；

其次，在以上数列中取11个6a-5类型的连续数（避开或包含前面筛选掉的那个6a-5类型的数都可以），以11为模，对不能使得

6a≡5(mod13)成立的6a-5类型的数进行同余分类，也只有1个同余类能够使得6a≡5(mod11)成立，筛选掉，其它10个同余类都不能使得

6a≡5(mod11)成立，纵使前面已经筛选掉的那个同余的6a-5类型的数也算一个以11为模的同余类，还有9个同余类都不能使得6a≡5(mod11)成立；

再次，以7为模，在上述数列中取7个6a-5类型的连续数，纵使不避开前面已经筛选掉的两个6a-5类型的数，对不能使得6a≡5(mod11)成立的6a-5类型的数进行同余分类，也只有1个同余类能够使得6a≡5(mod7)成立，筛选掉，其它的同余类都不能使得6a≡5(mod7)成立，也纵使前面已经筛选掉的两个同余的6a-5类型的数，在以7为模的同余分类中也各占一个同余类，至少还有4个同余类都不能使得6a≡

5(mod7)成立；

最后，以 5 为模，在上述数列中取 5 个连续的 6a-5 类型的数，纵使都不避开前面已经筛选掉的 3 个 6a-5 类型的数，还有两个同余类，同样最多只有 1 个同余类能够使得 6a≡5(mod5)成立，筛选掉，至少还有 1 个同余类不能使得 6a≡5(mod5)成立，即在 71～142 之间至少有 1 个 6a-5 类型的素数。

同理可证明：在 71～142 之间 11 个 6b-7 类型的连续数中，至少有 1 个不能使得 6b≡7(modP_j)成立的 6b-7 类型的素数。

综上所述，当 P_A=71 时，在 P_A～2P_A之间至少有 1 个 6a-5 类型的素数和 1 个 6b-7 类型的素数，即至少共有两个素数（实际上多达 14 个），定理 2 成立。

B、当 P_A>71，b>13，在 P_A～2P_A之间有 b-1 个 6a-5 类型的连续数和 b-2 个 6b-7 类型的连续数。

假设 b=u，u>13，在 6u-7～2(6u-7)之间有 u-1 个 6a-5 类型的连续数和 u-2 个 6b-7 类型的连续数，u 以内大于 3 的素数为 P_j，设 P_u为 P_j的最大值，P_u可能的最大值是 P_u=u 。

首先以 P_u为模，对 u-1 个 6a-5 类型的连续数进行同余分类，根据定理 1，最多只有 1 个同余类能够使得 6a≡5(modP_u)成立，筛选掉，其它 u-2 个同余类都不能使得 6a≡5(modP_u)成立；然后以 P_{u-1}为模，对那些不能使得 6a≡5(modP_u)成立的 6a-5 类型的数进行同余分类，最多也

只有 1 个同余类能够使得 $6a\equiv5(\mathrm{mod}P_{u-1})$ 成立，筛选掉，其它同余类都不能使得 $6a\equiv5(\mathrm{mod}P_{u-1})$ 成立；如此继续分别依次以 P_{u-2}、P_{u-3}……P_3、P_2 为模，对以前一个模不同余的 6a-5 类型的数进行同余分类，纵使每次都有 1 个同余类能够使得 $6a\equiv5(\mathrm{mod}P_j)$ 成立，逐步筛选掉，最后至少还有 1 个同余类不能使得 $6a\equiv5(\mathrm{mod}P_j)$ 成立，即至少有 1 个 6a-5 类型的数为素数。

而且，以 P_u 为模，对 u-2 个 6b-7 类型的连续数进行同余分类，也最多只有 1 个同余类能够使得 $6b\equiv7(\mathrm{mod}P_u)$ 成立，筛选掉，其它同余类都不能使得 $6b\equiv7(\mathrm{mod}P_u)$ 成立；然后以 P_{u-1} 为模，对那些不能使得

$6b\equiv7(\mathrm{mod}P_u)$ 成立的 6b-7 类型的数进行同余分类，最多也只有 1 个同余类能够使得 $6b\equiv7(\mathrm{mod}P_{u-1})$ 成立，筛选掉，其它同余类都不能使得

$6b\equiv7(\mathrm{mod}P_{u-1})$ 成立；如此继续分别依次以 P_{u-2}、P_{u-3}……P_3、P_2 为模，对以前一个模不同余的 6b-7 类型的数进行同余分类，也纵使每次都有 1 个同余类能够使得 $6b\equiv7(\mathrm{mod}P_j)$ 成立，逐步筛选掉，最后至少有 1 个同余类不能使得 $6b\equiv7(\mathrm{mod}P_j)$ 成立，即至少有 1 个 6b-7 类型的数为素数。

当 b=u+1 时，在 6(u+1)-7～2[6(u+1)-7]之间，有 u 个 6a-5 类型的连续数和 u-1 个 6b-7 类型的连续数，u+1 以内大于 3 的素数 P_j 可能与 b=u 时一样多，最多也只能比 b=u 增加 1 个，纵使为后者，设最大的 P_j 为 P_{u+1}，P_{u+1} 可能的最大值为 u+1。

以 P_{u+1} 为模，对 u 个 6a-5 类型的连续数进行同余分类，最多只有 1

个同余类能够使得 $6a\equiv5(\mathrm{mod}P_{u+1})$ 成立，筛选掉，其它 u-1 个同余类都不能使得 $6a\equiv5(\mathrm{mod}P_{u+1})$ 成立；然后完全可以与假设前提一样，继续分别依次以 P_u、P_{u-1}……P_3、P_2 为模，对以前一个模不同余的 6a-5 类型的数进行同余分类，也纵使每次都有 1 个同余类能够使得 $6a\equiv5(\mathrm{mod}P_j)$ 成立，逐步筛选掉，最后至少有 1 个同余类不能使得 $6a\equiv5(\mathrm{mod}P_j)$ 成立，即至少有 1 个 6a-5 类型的数为素数。

同样，以 P_{u+1} 为模，对 u-1 个 6b-7 类型的连续数进行同余分类，根据定理 1，也最多只有 1 个同余类能够使得 $6b\equiv7(\mathrm{mod}P_{u+1})$ 成立，筛选掉，其它 u-2 个同余类都不能使得 $6b\equiv7(\mathrm{mod}P_{u+1})$ 成立；然后也完全可以与假设前提一样，继续分别依次以 P_u、P_{u-1}……P_3、P_2 为模，对以前一个模不同余的 6b-7 类型的数进行同余分类，也纵使每次都有 1 个同余类能够使得 $6b\equiv7(\mathrm{mod}P_j)$ 成立，逐步筛选掉，最后至少有 1 个同余类不能使得 $6b\equiv7(\mathrm{mod}P_j)$ 成立，即至少有 1 个 6b-7 类型的数为素数。

总之，不管 P_A 为 6a-5 类型的素数，还是 6b-7 类型的素数，在 P_A～$2P_A$ 之间都至少有 1 个 6a-5 类型的素数和 1 个 6b-7 类型的素数，即至少有两个素数。定理 2 证明完毕。

定理 2 也可以表述为与其等价的定理 3。

定理 3、设整数 B>5，则在 B～2B 之间至少有两个素数。

证明：B 只有三种情况：当 B=6 时，2B=12，在 6～12 之间有 7 和 11 两个素数；

当 B 为大于 5 的素数 P_A 时，定理 3 即为定理 2，在 B～2B 之间至少有两个素数；

根据定理 2，在大于 5 的素数 P_A～$2P_A$ 之间至少有两个素数 P_{A+1} 和 P_{A+2}。显然，当 B 在 P_A～P_{A+1} 之间时，由于 $B<P_{A+1}<P_{A+2}$，而 $B>P_A$，即 $2B>2P_A$，所以在 B～2B 之间至少有两个素数 P_{A+1} 和 P_{A+2}。定理 3 证明完毕。

定理 4、设大于 4 的偶数为 2N，则 N 以内的奇素数 P_N，以 $(2N)^{1/2}$ 之内的任意一个奇素数 P_i 为模，至少可以分为 i+1 个同余类。

分析：设大于 4 的偶数为 2N，N 以内的奇素数为 P_N，$(2N)^{1/2}$ 之内的奇素数为 P_i，i 为 $(2N)^{1/2}$ 之内奇素数 P 的序数，i=1，2，3，…，n，即 P_i 的最大值为 P_n，n 为 i 的最大值，那么，P_N 的个数 $\pi(P_N)$ 为：当 2N=6～8 时，P_n 不存在，即 n=0，而 $\pi(P_N)=1$，所以 $\pi(P_N)=n+1$；当 2N=10～24 时，$P_n=3$，n=1，$\pi(P_N)=2$～4，$\pi(P_N)\geqslant n+1$；当 2N=26～34 时，$P_n=5$，n=2，$\pi(P_N)=5$～6，$\pi(P_N)>n+1$。

当 2N≥36 时，N≥18，而 $(2N)^{1/2}\geqslant 6$，即 $2(2N)^{1/2}\geqslant 12$，则 $N>2(2N)^{1/2}$。根据定理 3，设整数 B>5，则在 B～2B 之间至少有两个素数，即在 $(2N)^{1/2}$～$2(2N)^{1/2}$ 之间至少有两个素数，也即在 $(2N)^{1/2}$～N 之间至少有两个素数。因为在 $(2N)^{1/2}$ 之内的奇素数为 P_i，设在 $(2N)^{1/2}$～N 之间前面的两个奇素数依次为 P_{i+1} 和 P_{i+2}，所以在 N 之内至少有 n+2 个奇素数 P_N，即 $\pi(P_N)>n+1$。

综上所述，当 2N>4 时，总是 $\pi(P_N)\geqslant n+1$，显然 $\pi(P_N)\geqslant i+1$。

证明：小于 P_i 的 P_1、P_2…… P_{i-1} 奇素数，对模 P_i 来说，都是余数。它们的数值各不相等，所以都不同余。

$P_i|P_i$，P_i 显然与小于 P_i 的奇素数都不同余。

设大于 P_i 的相邻素数为 P_{i+1}，则 $P_{i+1}=P_i+r_1$，$r_1<P_i$。显然 P_{i+1} 与 P_i 不同余。P_{i+1} 与小于 P_i 的奇素数是不是同余呢?很明显，r_1 为偶数，P_{i+1} 与小于 P_i 的奇素数都不同余。

综上所述，以 P_i 为模，从 P_1 到 P_{i+1}，i+1 个 P_N 素数都不同余，所以 P_N 素数至少可以分为 i+1 个同余类。定理 4 证明完毕。

定理 5、设任意一个大于 4 的 2N 偶数，在 $(2N)^{1/2}$ 之内的奇素数为 P_i，该偶数的中数 N 以内的奇素数为 P_N，则必然存在以 P_i 为模而与 2N 偶数不同余的 P_N 素数 P_b。

证明：当 2N 偶数一经确定，偶数的中数 N 也随之确定，在 $(2N)^{1/2}$ 之内的奇素数 P_i 也被限定，N 以内的奇素数 P_N 也同样被限定，每个 P_i 去除 2N 偶数的余数也就一定。

当 2N=6～8 时，在 $(2N)^{1/2}$ 以内只有一个素数 2，即大于 2 的奇素数 P_i 还不存在，i=0，在 N 以内有一个 P_N 素数 3，即 $\pi(P_N)=1$，这个 P_N 素数不用进行同余分类筛法的筛选，显然为 P_b 素数，P_b 素数的个数 $\pi(P_b)=1$。

当 2N=10～24 时，在（2N）$^{1/2}$ 之内只有一个 P_i 素数 3，最大的 P_i 素数 $P_n=3$，在 N 以内的 P_N 素数开始有 3 和 5 两个，然后增加到有 3、5、7

三个，最后增加到有 3、5、7、11 四个，以 3 为模，P_N素数至少可以分为两个同余类，纵使有一个同余类与 2N 偶数同余，筛选掉，至少还有一个同余类与 2N 偶数不同余，P_b素数存在。

当 2N=26～34 时，在（2N）$^{1/2}$之内有两个 P_i素数 3 和 5，P_n=5, 在 N 以内的 P_N素数开始有 3、5、7、11、13 五个，最后增加到 3、5、7、11、13、17 六个。首先以 P_n素数 5 为模，可以将 P_N素数分为四个同余类，纵使有一个同余类与 2N 偶数同余，筛选掉，还有三个同余类与 2N 偶数不同余。然后以 P_i素数 3 为模，对以 5 为模而与 2N 偶数不同余的三个同余类中的 P_N素数进行同余分类，至少可以分为两个同余类，也纵使有一个同余类与 2N 偶数同余，筛选掉，至少还有一个同余类与 2N 偶数不同余。这个与 2N 偶数不同余的同余类中的 P_N素数即为 P_b素数。

当 2N≥36 时，N≥18, 而（2N）$^{1/2}$≥6, 即 2（2N）$^{1/2}$≥12, 则

N>2（2N）$^{1/2}$, P_i素数的个数 π（P_i）≥2, 根据定理 3，在（2N）$^{1/2}$～

2（2N）$^{1/2}$之间至少有两个素数 P_{i+1}和 P_{i+2}, 即在（2N）$^{1/2}$～N 之间至少有两个素数 P_{i+1}和 P_{i+2}, 也即在 N 以内奇素数的个数 π（P_N）≥i+2。因此，根据定理 4，运用同余分类筛法和归纳法有：

1、当 i=3 时，即(2N)$^{1/2}$之内有三个 P_i素数 3、5、7，2N=50～120, P_N素数至少有五个：3、5、7、11、13。

首先以 7 为模，至少能将 P_N 素数分为五个同余类，纵使有一个同余类与 2N 偶数同余，筛选掉，至少还有四个同余类与 2N 偶数不同余。

然后以 5 为模，对以 7 为模而与 2N 偶数不同余的四个同余类中的 P_N 素数进行同余分类，至少可以分为三个同余类，也纵使有一个同余类与 2N 偶数同余，筛选掉，至少还有两个同余类与 2N 偶数不同余。

最后以 3 为模，对以 5 为模而与 2N 偶数不同余的两个同余类中的 P_N 素数进行同余分类，至少可以分为两个同余类，纵使也有一个同余类与 2N 偶数同余，筛选掉，最后至少还有一个同余类与 2N 偶数不同余，P_b 素数存在。

2、假设 i=K，K>3，即 $(2N)^{1/2}$ 之内有 K 个 P_i 素数，最大的 P_i 素数为 P_K，以 P_K 为模，至少可以将 P_N 素数分为 K+1 个同余类，纵使有一个同余类与 2N 偶数同余，筛选掉，至少还有 K 个同余类与 2N 偶数不同余；然后继续分别依次用 P_{K-1}、P_{K-2}……P_2、P_1 为模，对以前一个 P_i 素数为模而与 2N 偶数不同余的同余类中的 P_N 素数进行同余分类，也纵使每次总有一个同余类与 2N 偶数同余，逐步筛选掉，最后至少还有一个同余类与 2N 偶数不同余，P_b 素数存在。

当 i=K+1 时，即 $(2N)^{1/2}$ 之内有 K+1 个 P_i 素数，最大的 P_i 素数为 P_{K+1}，以 P_{K+1} 为模，至少可以将 P_N 素数分为 K+2 个同余类，纵使有一个同余类与 2N 偶数同余，筛选掉，至少还有 K+1 个同余类与 2N 偶数不同余。

当以 P_K 为模时，P_1、P_2……P_{K-2}、P_{K-1} 均为奇素数的余数，P_K 的余数为

0，P_{K+1}的偶余数 r_1<P_{K+2}的偶余数 r_2。因此，无论上述哪一个同余类被筛选掉，与归纳假设完全一致，至少可以将以 P_{i+1}为模而与 2N 偶数不同余的 K+1 个同余类中的 P_N素数分为 K+1 个同余类，纵使也有一个同余类与 2N 同余，筛选掉，至少还有 K 个同余类与 2N 偶数不同余；然后继续分别依次用 P_{K-1}、P_{K-2}……P_2、P_1为模，对以前一个 P_i素数为模而与 2N 偶数不同余的同余类中的 P_N素数进行同余分类，也纵使每次总有一个同余类与 2N 偶数同余，逐步筛选掉，最后至少还有一个同余类与 2N 偶数不同余，P_b素数存在。

综上所述，当 2N>4，都存在以 P_i为模而与 2N 偶数不同余的 P_N素数 P_b。定理 5 得到了证明。

实际上，随着 2N 偶数的增大，P_N逐渐增大，$\pi(P_N)$不断增多，而且因为$\pi(P_N)$随 N 的增大而增多，即随 2N 的增大而增多，但$\pi(P_i)$只随$(2N)^{1/2}$的增大而增多，所以$\pi(P_N)$的增多比$\pi(P_i)$的增多要多得多；随着 2N 偶数越来越大，$\pi(P_N)$的增多比$\pi(P_i)$的增多也越来越多。因此，以 P_i为模，较大偶数的 P_N素数的同余分类，都多于 i+1 个而趋向 P_i个。例 1 将表明，50 就可以代表较大的偶数，P_N素数的同余分类基本完整，在整个同余分类筛选过程中，只有一个同余类缺失，P_b素数有多个。当 2N 偶数足够大时，P_N素数都可以分为 P_i个同余类（除了作为某一个 P_i模的那个 P_N素数已被前面筛选掉而使 0 同余类缺失以外）。例 2 将表明，100 就可以代表足够大的偶数了，P_N素数在整个同余分类筛选过程中，

都可以完整地分为P_i个同余类，P_b素数的个数更多。系统而大量的试算表明，在大于4的偶数中，仅有6、8、12的P_b素数只有一个，其它偶数都有两个或两个以上的P_b素数；随着2N偶数的增大，$\pi(P_b)$的下限将在1～2～3的基础上逐渐增加，上限更将在1～3～5～12～13的基础上不断增加，2N偶数越大，P_b素数越多。定理5证明完毕。

根据定理5，因为P_b是以P_i为模而与2N偶数不同余的P_N素数，即

$2N \equiv P_b \pmod{P_i}$不能成立，也就是说P_i不能整除$2N-P_b$，所以$2N-P_b$为素数。设$2N-P_b=q$，q为素数，则大于4的2N偶数都有一般的“1+1”解：

$$2N=P_b+q$$

即 $$2N=P_b+(2N-P_b)\ldots\ldots\ldots\ldots(1)$$

例1、求50的“1+1”解。

解：50的中数为25，P_i为3、5、7，P_N素数有3、5、7、11、13、17、19、23等8个。

第一步：求P_b素数：

1、以7为模，P_N素数可以分为六个同余类，刚好是1同余类没有P_N素数，没有与50同余的P_N素数类，不用筛选。

2、以5为模，将以7为模而与50不同余的六个同余类中的P_N素数分为完整的五个同余类，筛选掉与50同余的0同余类P_N素数5。

3、以3为模，将以5为模而与50不同余的四个同余类中的P_N素数分为完整的三个同余类，筛选掉与50同余的2同余类P_N素数11、17、

23，保留下来的两个同余类的P_N素数即为P_b素数，有3、7、13和19四个。

第二步：求“1+1”解：

将50代入(1)式的2N，将以上求出的50的P_b素数分别代入(1)式的P_b，即得50的“1+1”解：50=3+47，7+43，13+37，19+31。

50偶数的P_N素数以P_i为模，同余分类接近完整，在整个同余分类筛选过程中，只有一个同余类缺失，有多个“1+1”解，可以代表一般较大的偶数。

例2、求100的“1+1”的解。

解：100的中数为50，P_i为3、5、7，P_N素数有3、5、7、11、13、17、19、23、29、31、37、41、43、47等14个。

第一步：求P_b素数：

1、以7为模，将P_N素数分为完整的七个同余类，筛选掉与100同余的2同余类的P_N素数23和37。

2、以5为模，将以7为模而与100不同余的六个同余类中的P_N素数分为完整的五个同余类，筛选掉与100同余的0同余类的P_N素数5。

3、以3为模，将以5为模而与100不同余的四个同余类中的P_N素数分为完整的三个同余类，筛选掉与100同余的1同余类的P_N素数7、13、19、31和43，保留下来的两个同余类的P_N素数即为P_b素数，有3、11、17、29、41和47六个。

第二步：求“1+1”解：

将 100 代入(1)式的 2N，将以上求出的 100 的 P_b 素数分别代入(1)式的 P_b，即得 100 的“1+1”解：

100=3+97，11+89，17+83，29+71，41+59，47+53。

100 偶数的 P_N 素数以 P_i 为模，在整个同余分类筛选过程中，都可以分为完整的 P_i 个同余类，其“1+1”解多达六个，可以代表“足够大的偶数”了。

综合上述，4 有特定的“1+1”解，大于 4 的偶数都有一般的“1+1”解，而且偶数越大，“1+1”的解数趋于越多。 命题(1)得到了证明。

三、命题(2)的证明

命题(2)：每个大于 5 的奇数都是三个素数之和。

证明：命题(2)实际上是命题(1)的推论，所以命题(1)得到了证明，则命题(2)容易证明：一般地说，设奇数 M 大于 5，S 为素数，M-2>S>2，则 M-S 为大于 2 的偶数。

若 M-S=4，则 M-S=2+2，

即　$M=2+2+S$ .... (2-1)

若 M-S 为大于 4 的偶数 2N，根据(1)式，有

$$M-S=P_b+(2N-P_b),$$

即　$M=P_b+(2N-P_b)+S$ .... (2-2)

当 M 大于 11 时，(2-2)式都有多解性。

根据(2-1)和(2-2)式，当M大于7时，命题(2)都有多解性。M越大，解越多。

例3、求15是三个素数之和的解。

解：设S为素数，根据S的限定条件15-2>S>2，S有3、5、7、11四个，按照公式(2-1)和(2-2)，去掉相同的解，有15=7+5+3，5+5+5，2+2+11。

综合上述，命题(2)得到了证明。

四、结　　论

命题(1)的证明：

偶数4的“1+1”解为：4=2+2。

以一定素数为模，对一定类型的数进行同余分类，然后把对一定素数模0同余的同余类筛选掉的方法，叫做同余分类筛法。运用同余分类筛法，根据下列五条定理，便可证明每个大于4的偶数都有一般的“1+1”解。

定理1、设P_c为大于3的素数，在6a-5或6b-7类型数的数列中，任意取P_c个以内的连续数，则其中最多只有一个数能够使得$6a\equiv5(\mathrm{mod}P_c)$或$6b\equiv7(\mathrm{mod}P_c)$成立，其它各个数都不能使得$6a\equiv5(\mathrm{mod}P_c)$或

$6b\equiv7(\mathrm{mod}P_c)$成立。

定理2、设P_A为大于5的一定素数，则在$P_A\sim2P_A$之间至少有两个

素数。

定理 2 也可以表述为与其等价的定理 3。

定理 3、设整数 B>5，则在 B～2B 之间至少有两个素数。

定理 4、设大于 4 的偶数为 2N，则 N 以内的奇素数 P_N，以 $(2N)^{1/2}$ 之内的任意一个奇素数 P_i 为模，至少可以分为 i+1 个同余类。

定理 5、设任意一个大于 4 的 2N 偶数，在 $(2N)^{1/2}$ 之内的奇素数为 P_i，该偶数的中数 N 以内的奇素数为 P_N，则必然存在以 P_i 为模而与 2N 偶数不同余的 P_N 素数 P_b。

根据定理 5，大于 4 的 2N 偶数都有一般的“1+1”解：

$2N=P_b+(2N-P_b)$。

综合 4 与大于 4 的偶数都有“1+1”解，命题(1)得到了证明。

命题(2)的证明：设奇数 M 大于 5，S 为素数，M-2>S>2，则 M-S 为大于 2 的偶数。

若 M-S=4，则 M-S=2+2，

即　M=2+2+S；

若 M-S 为大于 4 的偶数 2N，根据(1)式有

$$M-S=P_b+(2N-P_b),$$

即　$M=P_b+(2N-P_b)+S$。

命题(2)得到了证明。

命题(1)和(2)都得到了一般的证明，则哥德巴赫猜想完全得到了一

般的证明。哥德巴赫猜想今后应当改称为哥德巴赫定理。

说明：

① 命题(1)又称命题“1+1”，为了简便，下文将引用。

②含偶数的中数。下同。

注释：

[1]辞海，上海辞书出版社，缩印本。 1989年版。

[2]华罗庚，数论导引，科学出版社，1979，P.90。

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THE PROOF OF GOLDBACH CONJECTURE BY SIEVE METHOD OF CONGRUENCE CLASSIFICATION*

ABSTRACT: The author has pioneered an innovative sieve method of Congruence Classification first of all in the paper, and then he has proved five innovative theorems. He has proved generally at first, the Proposition(1) of Goldbach conjecture: Each even number more than 2 is the sum of two prime numbers, and then the Proposition (2):Each odd number more than 5 is the sum of three prime numbers. Goldbach conjecture has been proved generally completely.

Key Words: Goldbach conjecture, Sieve method of congruence classification, Five innovative theorems, Prove generally.

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1 INTRODUCTION

Goldbach conjecture, put forward in the letter which Christian Goldbach (German mathematician) wrote to Leonhard Euler on 7th June, 1742, is one of the well-known problems in the theory of number; Due to 1 is prime number too at that time, he advances two propositions:(1) Each even number is the sum of two prime numbers; (2) Each odd number more than 1 is the sum of three prime numbers. Afterwards, International Congress of Mathematicians has been decided that 1 is not prime number, thus, two propositions of Goldbach Conjecture ought to rewritten as: (1) Each even number more than 2 is the sum of two prime numbers①; (2) Each odd number more than 5 is the sum of three prime numbers [1]. In the past more than 200 years, many mathematicians in China and other countries made a great effort to prove this conjecture. But, any general proof of this conjecture has not yet been obtained. The general proof of this conjecture is really complicated .The author began to make a study of it in 1978. It is 17 years since he researched into this problem. And he spent 5 years to finalize his thesis. The author has pioneered an innovative sieve method of congruence classification first of all, and then he has proved five innovative theorems. He has proved generally that Goldbach Conjecture is tenable completely.

2 PROOF OF PROPOSITION (1)

Proposition (1): Each even number more than 2 is the sum of two prime numbers.

In order to prove Proposition (1), a systematic study must be begun from integer. Taking 2 as module, integer is divided into two classes of even numbers and odd numbers. Excepting for 2, as a prime number, all other even numbers are composite numbers .Odd number includes all prime numbers more than 2 and many composite numbers.

4 is a special case in even numbers more than 2, and it is formed by even prime number 2 into 2P-type (P is prime number) even number. The solution of "1+1": 4=2+2.

As for even numbers more than 4, prime number 2 can' t consist of solution of "1+1". Therefore, when the solution of "1+1"for even numbers more than 4 is studied, prime number 2 is not necessary to be considered essentially, and the consideration can be made only in odd prime numbers more than 2.

There are odd prime numbers in integers more than 2 at any interval (i. e. within mean numbers of even numbers more than 4②), and the quantity of odd prime numbers is indefinite [2]. This provides the precondition for proving that all even numbers more than 4 have the general solution of "1+1".

In order to prove that even number 2N more than 4 have the general solution of "1+1", it is necessary to take definite prime number as module, and divide the definite type numbers into congruence classes, then sieve the congruence class congruent to module 0 of definite prime numbers. This method of sieve based on congruence classification is

called as the sieve method of congruence classification. According to the following five theorems, the sieve method of congruence classification can be used to prove that each even number more than 4 has the general solution of "1+1".

Taking 6 as module, the integral number progression is divided into congruence classes, and the prime numbers more than 3 can appear only in two congruence classes of $6m \pm 1(m>0)$, i .e. these prime numbers can be formulated as $6a-5(a>1)$ type or $6b-7(b>1)$ type numbers. However, Theorem 1 is given for the two type numbers stated above.

Theorem 1: If P_c is prime number more than 3, the continuous numbers within P_c are taken optionally from $6a-5$ type or $6b-7$ type number progression, then only one number at most in these continuous numbers can render that $6a\equiv5(\text{mod } P_c)$ or $6b \equiv 7(\text{mod } P_c)$ is tenable, but all other numbers can not render that $6a\equiv5(\text{mod } P_c)$ or $6b\equiv7(\text{mod } P_c)$ are tenable .

Demonstration: All $6a-5$ type and $6b-7$ type number progressions are the arithmetical series whose equal difference is 6. In $6a-5$ type and $6b-7$ type number progressions, as $2|6a$, $2|6b$ but 2 can not divide exactly 5, 2 can not divide exactly 7, thus 2 can not divide exactly $6a-5$, 2 can not divide exactly $6b-7$; Similarly, 3 can not divide exactly $6a-5$, 3 can not divide exactly $6b-7$. When adjudging whether $6a-5$ type or $6b-7$ type numbers are prime numbers, therefore, it is necessary for modules not to take account of prime number 2 and 3 but prime number P_c more than 3 only.

Given an optional $6a-5$ $(a>1)$ type number, P_c arithmetical series whose continuous equal difference following it is 6 are $6(a+1)-5$, $6(a+2)-5$, $6(a+3)-5\cdots\cdots6(a+P_c-2)-5$, $6(a+P_c-1)-5$, $6(a+P_c)-5$. In the progressions of this arithmetical series, if a number $6(a+m)-5$ $(0<m\leqslant P_c)$ divided by prime number P_c more than 3 is integer, i. e. $6(a+m)-5=nP_c$ $(n>0)$, other P_c-1 numbers are $6(a+m)-5\pm6e$ $(0<e<P_c$, e is integer). Which divided by P_c is $[6(a+m)-5]/P_c \pm 6e/P_c$. As $P_c \mid 6(a+m)-5$ but 6e not to include the factor of P_c, P_c can not divide exactly 6e, thus P_c can not divide exactly $6(a+m)-5\pm6e$—each number has always definite remainder number r $(0<r<P_c$, r is integer). Because this progression is the arithmetical series, it is impossible that the remainder of each number is equal. The varied value of remainder number is $1\sim P_c-1$, but the number with remainder is P_c-1 also, thus the remainder of each number inevitably is one number among $1,2,3\cdots\cdots P_c-3$, P_c-2 ,P_c-1 respectively. The distribution sequence of these remainders varies with starting number, but their distribution law is definite.

The starting number of arithmetical series whose P_c equal difference in the continuous numbers above are 6 can be determined optionally. For example: $6(a+P_c-2)-5$, $6(a+P_c-1)-5$, $6(a+P_c)-5$, $6(a+P_c+1)-5\cdots\cdots6(a+2P_c-4)-5$ and $6(a+2P_c-3)-5$ etc.

Similarly, it can be proved that only one number $6(b+m)-7(0<m\leqslant P_c)$ within P_c arithmetical series whose equal differences in the continuous numbers following an optional $6b-7$ $(b>1)$ are 6 can. be divided by P_c, with no remainder, to make $6(b+m)-7$ equal nP_c $(n>0)$, but other Pc−1 numbers divided by P_c are integer impossibly, with definite remainder r $(0<r<P_c)$ after all. Within P_c-1 arithmetical series, the remainder of

each number is inevitably one number among 1,2,3 ……P_c –3, P_c–2, P_c–1 respectively. Also, the distribution law of these remainders is always definite. Meanwhile, the starting number of arithmetical series 6b–7 whose P_c equal differences in the continuous numbers are 6 can be determined optionally.

Since in the progression of 6a –5 type or 6b –7 type arithmetical series only one number can render that $6a \equiv 5 (\bmod P_c)$ or $6b \equiv 7 (\bmod P_c)$ is tenable and all other $P_c -1$ numbers can not render that $6a \equiv 5 (\bmod P_c)$ or $6b \equiv 7 (\bmod P_c)$ are tenable when P_c continuous numbers exist, obviously , only one number at most can render that $6a \equiv 5 (\bmod P_c)$ or $6b \equiv 7 (\bmod P_c)$ is tenable and all other numbers can not render that $6a \equiv 5 (\bmod P_c)$ or $6b \equiv 7 (\bmod P_c)$ are tenable when only $2 \sim P_c-1$ continuous numbers exist in the progression of 6a-5 type or 6b–7 type numbers. Theorem 1 has been proved.

Theorem 2: If P_A is the definite prime number more than 5,two prime numbers at least exist within $P_A \sim 2P_A$

Analysis: In order to facilitate the proof of Theorem 2, it is necessary to know at firs the continuous distribution of 6a-5 type and 6b –7 type numbers between $P_A \sim 2P_A$ and the relation of a with $[2(6a-5)]^{1/2}$ and of b with $[2(6b-7)]^{1/2}$.

1.The Continuous Distribution of 6a-5 Type and 6b-7 Type Numbers Between $P_A \sim 2P_A$.

(1) When P_A is 6a-5 type prime number:

① If x is integer more than O, 6(a+ x)-5 type numbers (not sure to be prime numbers) between $P_A \sim 2P_A$ are $6a-5<6(a+x)-5<2(6a-5)$.

From $6a-5<6(a+x)-5$, we have

$$6a-5<6a+6x-5$$
$$6x>0$$
$$\text{i.e. } x>0$$

From $6(a+x)-5<2(6a-5)$, we have

$$6a+6x-5<12a-10$$
$$6x<6a-5$$
$$x<a-5/6$$

When taking account of integer only, $x<a$.

As stated above, $0<x<a$. Therefore, a –1 continuous numbers of 6(a+ x)-5 type exist between $P_A \sim 2P_A$.

② 6b–7 type numbers (not sure to be prime number) between $P_A \sim 2P_A$ are $6a-5<6b-7<2(6a-5)$.

From $6a-5<6b-7$,we have

$$6b>6a+2$$
$$b>a+1/3$$

When taking account of integer only, $b>a$.

From $6b-7<2(6a-5)$, we have

$$6b-7<12a-10$$
$$b<2a-1/2$$

When taking account of integer only, $b<2a$.

As stated above, $a<b<2a$. Thus, $a-1$ continuous numbers of $6b-7$ type exist between $P_A \sim 2P_A$ also.

(2) When P_A is $6b-7$type prime number:

① $6a-5$ type numbers between $P_A \sim 2P_A$ are $6b-7<6a-5<2(6b-7)$.

From $6b-7<6a-5$, we have

$$6a>6b-2$$
$$a>-b-1/3$$

When taking account of integer only, $a>b-1$.

From $6a-5<2(6b-7)$, we have

$$6a-5<12b-14$$
$$6a<12b-9$$
$$a<2b-3/2$$

When taking account of integer only, $a<2b-1$.

As stated above, $b-1<a<2b-1$. Thus, $b-1$ continuous numbers ($a=b,b+1,\cdots\cdots,2b-3,2b-2$ etc)of $6a-5$ type exist between $P_A \sim 2P_A$.

②If y is integer more than 0, $6(b+y)-7$ type numbers between $P_A \sim 2P_A$ are $6b-7<6(b+y)-7<2(6b-7)$.

From $6b-7<6(b+y)-7$, we have

$$6b-7<6b+6y-7$$
$$6y>0$$

i.e. $$y>0$$

From $6(b+y)-7<2(6b-7)$, we have

$$6b+6y-7<12b-14$$
$$6y<6b-7$$
$$y<b-7/6$$

When taking account of integer only, $y<b-1$.

As stated above, $0<y<b-1$. Therefore, $b-2$ continuous numbers of $6(b+y)-7$ type exist between $P_A \sim 2P_A$.

In a word, $a-1$ continuous numbers of both $6a-5$ type and $6b-7$ type exist between $P_A \sim 2P_A$ when P_A is $6a-5$ type prime number; $b-1$ continuous numbers of $6a-5$ type and $b-2$ continuous numbers of $6b-7$ type exist between $P_A \sim 2P_A$ when P_A is $6b-7$ type prime number. No matter what P_A is $6a-5$ type or $6b-7$ type prime number, one $6a-5$ type and one $6b-7$ type numbers increase between $P_A \sim 2P_A$,whenever a or b increases 1.

2.The Relation of a With $[2(6a-5)]^{1/2}$ and b With $[2(6b-7)]^{1/2}$.

(1) The relation of a with $[2(6a-5)]^{1/2}$.With increase of a, $[2(6a-5)]^{1/2}$ increases gradually but by small and small. Therefore, when a increases to the definite numerical value, we have

$$a>[2(6a-5)]^{1/2}$$
$$a^2>12a-10$$
$$a^2-12a+10>0$$

Resolving the inequalities stated above, we obtain

$$a>11.1$$

When taking account of integer only, i .e. as a>11,these inequalities are founded.

(2) The relation of b with $[2(6b-7)]^{1/2}$: Similarly, it can be resolved that Inequality $b>[2(6b-7)]^{1/2}$ is founded when b >10, by use of the above method that $a > [2(6a-5)]^{1/2}$ was proved to be founded when a >11.

The above calculations have shown that when a >11,$a>[2(6a-5)]^{1/2}$; when b >10, $b>[2(6b-7)]^{1/2}$.For the sake of easy use, no matter what P_A is 6a-5 type or 6b −7 type prime number, we can take one greater number as the criterion and determine them uniformly to be: when a>11 ,$a>(2P_A)^{1/2}$; when b>11 , $b>(2P_A)^{1/2}$. Therefore, when a >11 or b >11, prime numbers P_j more than 3 within a or b can be used to replace the prime numbers more than 3 within $(2P_A)^{1/2}$, then we adjudge whether 6a-5 type and 6b −7 type numbers between P_A~$2P_A$ are prime numbers.

Demonstration: By use of a>11 and b>11, P_A is divided into two parts of prime numbers both between 7~61 and more than 61. Also, the proof of Theorem 2 can be divided into two steps.

1. Prove that an optional prime number P_A between P_A~$2P_A$ in the progression of prime numbers 7~61 has two prime numbers at least of both P_{A+1} and P_{A+2}.

One prime number P_A between 7~61 is selected optionally. It can be seen directly from the very limited prime numbers table, and even many people having elementary knowledge of number theory can clearly remember, that there exist two prime numbers (P_{A+1} and P_{A+2}) at least between P_A~$2P_A$, thus the mathematical logic proof is not necessary to be done. In fact, only when P_A=7, there are only two prime numbers (11 and 13) between 7~14.When P_A =11~61, prime numbers between P_A~$2P_A$ increase from three to twelve obviously.

2. Prove that the prime number P_A more than 61 between P_A~$2P_A$ has two prime numbers (P_{A+1} and P_{A+2}) at least.

When P_A>61, i. e. a>11, b>11. In the progression of prime numbers more than 61, the least 6a-5 type prime number is 67, but the least 6b-7 type prime number is 71. By use of the sieve method of congruence classification and the inductive method, we have:

(1) When P_A is 6a-5 type prime number:

①According to the above description, when P_A =67(a=12), there are a-1 =11 continuous numbers of both 6a-5 type and 6b-7 type between P_A~$2P_A$ respectively. There include prime number 5,7,11 more than 3 within a.

According to Theorem 1, taking prime number 11 as module, eleven continuous numbers of 6a-5 type are divided into congruence classes, in which only one 6a-5 type numbers can render that $6a\equiv5 \pmod{11}$ is tenable and is sieved, thus other ten 6a-5 type numbers can not render that $6a\equiv5 \pmod{11}$ are tenable.

Secondly, taking seven continuous numbers of 6a-5 type from the above number progression, which 6a-5 type numbers of $6a\equiv5 \pmod{11}$ sieved above can be kept away or included, taking 7 as module and dividing them into congruence classes, only one 6a-5

type number can render that $6a\equiv5 \pmod 7$ is tenable and is sieved; even if 6a-5 type number sieved above is included, there are still five 6a-5 type numbers at least which can not render that $6a\equiv5 \pmod 7$ are tenable.

At last, taking five continuous numbers of 6a-5 type from the above number progression ,which 6a-5 type numbers of both $6a\equiv5 \pmod{11}$ and $6a\equiv5 \pmod 7$ sieved above can be kept away or included, and taking 5 as module, they are divided into congruence classes; even if the congruent 6a-5 type numbers are not repeated, only one 6a-5 type number can render that $6a\equiv5 \pmod 5$ is tenable and is sieved; with the exception of two 6a-5 type numbers sieved above, two 6a-5 type numbers at least can not render that $6a\equiv5 \pmod 5$ are tenable and become prime numbers .

Similarly, it can be proved that two 6b–7 type numbers at least among eleven 6b –7 type continuous numbers between 67~134 can not render that $6b\equiv7$(mod 11,7,5) is tenable and become prime numbers.

As stated above, when P_A =67,there are two 6a-5 type and two 6b –7 type prim numbers at least between P_A~$2P_A$, i. e. there are at least four prime numbers in total (in fact, thirteen prime numbers). Thus, Theorem 2 is tenable.

②When P_A >67 (a>12), there are a-1 continuous numbers of both 6a-5 type and 6b –7 type between P_A~$2P_A$ respectively.

Given a =u (u is integer more than 12), there are u –1 continuous numbers of both 6a-5 type and 6b –7 type between 6u-5 ~2(6u- 5) respectively and the prime numbers more than 3 within u are P_j. If P_u is the maximal value of P_j, the possible maximum of P_u is P_u =u.

At first, taking P_u as module, u–1 continuous numbers of 6a-5 type are divided into congruence classes; according to Theorem 1 , only one congruence class at most can render that $6a\equiv5 \pmod{P_u}$ is tenable and is sieved, thus other u–2 congruence classes can not render that $6a\equiv5 \pmod{P_u}$ are tenable; and then taking P_{u-1} as module, 6a-5 type numbers [which can not render that $6a\equiv5 \pmod{P_u}$ are tenable] are divided into congruence classes; at this time, only one congruence class at most can render that $6a\equiv5 \pmod{P_{u-1}}$ is tenable and is sieved, thus other congruence classes can not render that $6a\equiv5 \pmod{P_{u-1}}$ are tenable; in this way, continuously taking P_{u-2},P_{u-3} ······P_3, P_2 as module in order respectively, 6a-5 type numbers not congruent to one module above are divided into congruence classes; even if one congruence class existing each time can render that $6a\equiv5 \pmod{P_j}$ is tenable and is sieved gradually, one congruence class at least can not render that $6a\equiv5 \pmod{P_j}$ is tenable finally, i. e. One 6a-5 type number at least is prime number.

And that taking P_u as module, u-1 continuous numbers of 6b –7 type are divided into congruence classes; at this time, only one congruence class at most can render that $6b\equiv7 \pmod{P_u}$ is tenable and is sieved, thus other u-2 congruence class can not render that $6b\equiv7 \pmod{P_u}$ are tenable; and then taking P_{u-1} as module, 6b-7 type numbers [which can not render that $6b\equiv7 \pmod{P_u}$ are tenable] are divided into congruence classes; at this time, only one congruence class at most can render that $6b\equiv7 \pmod{P_{u-1}}$ is tenable and is

sieved, thus other congruence classes can not render that $6b\equiv7(\mathrm{mod}\ P_{u-1})$ are tenable; in this way, continuously taking P_{u-2}, $P_{u-3}\cdots\cdots P_3$, P_2 as module in order respectively, 6b-7 type numbers not congruent to one module above are divided into congruence classes; even if one congruence class existing each time can render that $6b\equiv7(\mathrm{mod}\ P_j)$ is tenable and is sieved gradually, at last, one congruence class at least can not render that $6b\equiv7\ (\mathrm{mod}\ P_j)$ is tenable, i. e. One 6b-7 type number at least is prime number.

When a=u+1, there are u continuous numbers of both 6a-5 type and 6b –7 type between 6(u+1)-5~2[6(u+1)-5] respectively. Prime number P_j more than 3 within u+1 possibly is as many as that when a = u, but one prime number at most increases, compared with that when a=u; even if the latter appears, if P_j (max.) is P_{u+1}, the possible maximum of P_{u+1} is u+1.

Taking P_{u+1} as module, u continuous numbers of 6a-5 type are divided into congruence classes; at this time, only one congruence class at most can render that $6a\equiv5(\mathrm{mod}\ P_{u+1})$ is tenable and is sieved, thus u-1 congruence classes can not render that $6a\equiv5(\mathrm{mod}\ P_{u+1})$ are tenable; and then according to the hypothetical preconditions, continuously taking P_u, $P_{u-1}\cdots\cdots P_3,P_2$ as module in order respectively, 6a-5 type numbers not congruent to one module above are divided into congruence classes; even if one congruence class existing each time can render that $6a\equiv5(\mathrm{mod}\ P_j)$ is tenable and is sieved gradually, at last, one congruence class at least can not render that $6a\equiv5(\mathrm{mod}\ P_j)$ is tenable, i. e. One 6a-5 type number at least is prime number.

Similarly, taking P_{u+1} as module, u continuous numbers of 6b-7 type are divided into congruence classes; at this time, only one congruence class at most can render that $6b\equiv7(\mathrm{mod}\ P_{u+1})$ is tenable and is sieved, thus other u-1 congruence classes can not render that $6b\equiv7(\mathrm{mod}\ P_{u+1})$ is tenable; and then according to the hypothetical preconditions, continuously taking P_u, $P_{u-1}\cdots\cdots P_3,P_2$ as module in order respectively, 6b –7 type numbers not congruent to one module above are divided into congruence classes; even if one congruence class existing each time can render that $6b\equiv7(\mathrm{mod}\ P_j)$ is tenable and is sieved gradually , one congruence class at least can not render that $6b\equiv7(\mathrm{mod}\ P_j)$ is tenable finally, i. e. One 6b –7 type number at least is prime number.

As stated above, when P_A is 6a-5 type prime number more than 67, one 6a-5 type and one 6b-7 type prime number at least between P_A~$2P_A$ exist, i. e. two prime numbers at least exist totally.

(2) When P_A is 6b –7 type prime numbers:

① When P_A =71(b=13), b-1 =12 continuous numbers of 6a-5 type and b-2=11 continuous numbers of 6b-7 type between P_A~$2P_A$ exist. Prime number P_j more than 3 including within b is 5,7,11,13.

At first, taking 13 as module, twelve continuous numbers of 6a-5 type are divided into congruence classes; at this time, only one congruence class at most can render that $6a\equiv5(\mathrm{mod}\ 13)$ is tenable and is sieved, thus other eleven congruence classes can not render that $6a\equiv5(\mathrm{mod}\ 13)$ are tenable.

Secondly, taking eleven 6a-5 type continuous numbers (6a-5 type number sieved

above can be kept away or included) from the number progressions above, taking 11 as module, 6a-5 type numbers [which can not render that 6a-5≡(mod 13) are tenable] are divided into congruence classes; at this time, only one congruence class can render that 6a≡5(mod 11) is tenable and is sieved also, thus other ten congruence classes can not render that 6a≡5 (mod 11) are tenable; even if the congruence class sieved above is one congruence class taking 11 as module also, nine remainder congruence classes can not render that 6a≡5(mod 11) are tenable.

Once again, taking 7 as module, and taking seven 6a-5 type continuous numbers from the number progression above, if two 6a-5 type numbers sieved above are not kept away, 6a-5 type numbers [which can not render that 6a≡5(mod 11) are tenable] are divided into congruence classes, thus only one congruence class can render that 6a≡5(mod 7) is tenable and is sieved, thus other congruence classes can not render that 6a≡5 (mod 7) are tenable; even if two congruent 6a-5 type numbers sieved above have one congruence class respectively in the classification of congruence taking 7 as module, four congruence classes at least can not render that 6a≡5(mod 7) are tenable.

At last, taking 5 as module, and taking five continuous 6a-5 type numbers from the number progression above, even if three 6a-5 type numbers sieved above are not kept away, two congruence classes still exist; similarly, only one congruence class at most can render that 6a≡5(mod 5) is tenable and is sieved, thus one congruence class at least can not render that 6a≡5 (mod 5) is tenable, i. e. There is one 6a-5 type prime number at least between 71~142.

Similarly, it can be proved that there is one 6b –7 type prime number at least [which can not render that 6b≡7(mod P_j) is tenable] within eleven 6b –7 type continuous numbers between 71~142.

As stated above, when P_A =71, there are one 6a-5 type and one 6b –7 type prime number at least between P_A~2P_A, i.e. two prime numbers at least in total (in fact, up to fourteen prime numbers). Thus, Theorem 2 is tenable.

②When P_A >71 (b>13), there are b –1 continuous numbers of 6a-5 type and b–2 continuous numbers of 6b-7 type between P_A~2P_A.

Given b=u (u>13), there are u- 1 continuous numbers of 6a-5 type and u-2 continuous numbers of 6b-7 type between 6u-7~2(6u –7). The prime number more than 3 within u is P_j .If P_u is the maximum of P_j, the possible maximum of Pu is P_u =u.

Firstly, taking P_u as module, u-1 continuous numbers of 6a-5 type are divided into congruence classes. According to Theorem 1, only one congruence class at most can render that 6a≡5(mod P_u) is tenable and is sieved, thus other u –2 congruence classes can not render that 6a≡5(mod P_u) are tenable; and then taking P_{u-1} as module, 6a –5 type numbers [which can not render that 6a≡5(mod P_u) are tenable] are divided into congruence classes; at this time, only one congruence classes at most can render that 6a ≡5(mod P_{u-1}) is tenable and is sieved, thus other congruence classes can not render that 6a ≡5(mod P_{u-1}) are tenable. In this way, continuously taking P_{u-2}, P_{u-3}……P_3,P_2 as module in order respectively, 6a-5 type numbers not congruent to one module above are

divided into congruence classes; even if one congruence class existing each time can render that $6a\equiv5 \pmod{P_j}$ is tenable and is sieved gradually, at last, one congruence class at least can not render that $6a\equiv5 \pmod{P_j}$ is tenable, i. e. One 6a-5 type number at least is prime number.

Taking P_u as module further, u –2 continuous numbers of 6b –7 type are divided into congruence classes. At this time, only one congruence class at most can render that $6b\equiv 7(\bmod P_u)$is tenable and is sieved, thus other congruence classes can not render that $6b\equiv 7(\bmod P_u)$ are tenable; and then taking P_{u-1} as module, 6b-7 type numbers [which can not render that $6b\equiv7(\bmod P_u)$ are tenable] are divided into congruence classes; at this time, only one congruence class at most can render that $6a \equiv7 (\bmod P_{u-1})$ is tenable and is sieved, thus other congruence classes can not render that $6b\equiv7 (\bmod P_{u-1})$ are tenable. In this way, continuously taking P_{u-2} ,P_{u-3}……P_3,P_2 as module in order respectively, 6b–7 type numbers not congruent to one module above are divided into congruence classes; even if one congruence class existing each time can render that $6b\equiv 7(\bmod P_j)$ is tenable and is sieved gradually, at last, one congruence class at least can not render that $6b\equiv7(\bmod P_j)$ is tenable, i. e. One 6b–7 type number at least is prime number.

When b = u+1, there are u continuous numbers of 6a-5 type and u-1 continuous numbers of 6b-7 type between 6(u+1) –7~2[6(u +1)-7]. At this time, prime number P_j more than 3 within u+ 1 is as many as that when b=u possibly, but at most one prime number increases, compared with that when b = u; even if the latter appears, if P_j (max) is P_{u+1}, the possible maximum of P_{u+1} is u+1.

Taking P_{u+1} as module, u continuous numbers of 6a-5 type are divided into congruence classes; at this time, only one congruence class at most can render that $6a\equiv 5(\bmod P_{u+1})$ is tenable and is sieved, thus other u-1 congruence classes can not render that $6a\equiv5(\bmod P_{u+1})$are tenable; and then according to the hypothetical preconditions, continuously taking P_u , P_{u-1}……P_3,P_2 as module in order respectively, 6a-5 type numbers not congruent to one module above are divided into congruence classes; even if one congruence class existing each time can render that $6a\equiv5 (\bmod P_j)$ is tenable and is sieved gradually, at last, one congruence class at least can not render that $6a\equiv5 (\bmod P_j)$ is tenable, i. e. One 6a –5 type number at least is prime number.

Similarly, taking P_{u+1} as module, u–1 continuous numbers of 6b –7 type are divided into congruence classes; according to Theorem 1, only one congruence class at most can render that $6b\equiv7(\bmod P_{u+1})$ is tenable and is sieved, thus other u –2 congruence classes can not render that $6b\equiv7(\bmod P_{u+1})$ are tenable; and then according to the hypothetical preconditions, continuously taking P_u, P_{u-1}……P_3, P_2 as module in order respectively, 6b –7 type numbers not congruent to one module above are divided into congruence classes; even if one congruence class existing each time can render that $6b\equiv7(\bmod P_j)$ is tenable and is sieved gradually, at last, one congruence class at least can not render that $6b\equiv 7(\bmod P_j)$ is tenable, i. e. One 6b–7 type number at least is prime number.

In a word, no matter what P_A is 6a-5 type or 6b–7 type prime number, there are one

6a-5 type and one 6b–7 type prime number at least between P_A~$2P_A$, i. e. two prime numbers at least exist. Theorem 2 has been proved.

Also, Theorem 2 can be formulated as Theorem 3 equivalent to it.

Theorem 3: Given integer B>5, there are two prime numbers at least between B~2B.

Demonstration: B has three cases only. When B= 6 (2B=12), there are prime number 7 and 11 between 6~12.

When B is prime number P_A more than 5, Theorem 3 can be considered as Theorem 2, thus there are two prime numbers at least between B~2B.

According to Theorem 2, there are prime number P_{A+1} and P_{A+2} at least between prime number P_A~$2P_A$ more than 5. Evidently, when B is situated between P_A~P_{A+1},because $B<P_{A+1}<P_{A+2}$ but $B>P_A$, i. e. $2B>2P_A$, there are prime number P_{A+1} and P_{A+2} at least between B~2B. Theorem 3 has been proved.

Theorem 4: If even number more than 4 is 2N, odd prime number P_N within N, taking an optional odd prime number P_i within $(2N)^{1/2}$ as module, can be divided into i +1 congruence classes at least.

Analysis: If even number more than 4 is 2N,odd prime number within N is P_N, odd prime number within $(2N)^{1/2}$ is P_i, i is ordinal number of odd prime number P within $(2N)^{1/2}$, i=1,2,3……n, i. e. The maximal value of P_i is P_n (n is the maximal value of i), then number $\pi(P_N)$ of odd prime number P_N within N is that: When 2N=6~8, P_n is not existent, i. e. n=0, and $\pi(P_N)=1$, therefore $\pi(P_N)=n+1$; When 2N = 10~24, $P_n=3$, n=1, $\pi(P_N)=2$~4 , $\pi(P_N)\geqslant n+1$; When 2N =26~34, $P_n=5$, n=2, $\pi(P_N)=5$~6, $\pi(P_N)>n+1$; When 2N $\geqslant 36$, $N\geqslant 18$, and $(2N)^{1/2}\geqslant 6$, i. e. $2(2N)^{1/2}\geqslant 12$, then $N>2(2N)^{1/2}$. According to Theorem3, given integer B>5, there are two prime numbers at least between B~2B, i. e. there are two prime numbers at least between $(2N)^{1/2}$ ~$2(2N)^{1/2}$, i. e. there are two prime number at least between $(2N)^{1/2}$~N also. Because odd prime number within $(2N)^{1/2}$ is P_i, if two prime umbers at the head of prime number between $(2N)^{1/2}$~N is P_{i+1} and P_{i+2}, therefore there are n+2 odd prime numbers P_N at least within N, i. e. $\pi(P_N)>n+1$.

As stated above, when 2N>4, always $\pi(P_N)\geqslant n+1$.Odviously, $\pi(P_N)\geqslant i+1$.

Demonstration: In the case of module P_i, odd prime number P_1, P_2……P_{i-1} less than P_i are remainders. As their numerical values are not equal respectively, they are not congruence numbers.

Obviously, $P_i \mid P_i$, P_i and odd prime numbers less than P_i are not congruent.

Suppose any adjacent prime number more than P_i is P_{i+1}, then $P_{i+1}=P_i+r_1$, $r_1<P_i$. Clearly, P_{i+1} is not congruent to P_i. Is P_{i+1} congruent to odd prime numbers less than P_i? Evidently, when r_1 is an even number, P_{i+1} and odd prime numbers less than P_i are totally not congruent.

As stated above, when P_i is taken as module, all i +1 prime numbers P_N from P_1 to P_{i+1} are not congruent. Thus, prime number P_N can be divided into i+1 congruence classes at least. Theorem 4 has been proved.

Theorem 5: Given an optional even number 2N more than 4, odd prime number

within $(2N)^{1/2}$ is P_i, and odd prime number within mean number N of this even number is P_N, then P_b of prime number P_N taking P_i as module but not congruent to even number 2N exists inevitably.

Demonstration: When even number 2N is once determined, mean number N of even number is also determined. Meanwhile, odd prime number P_i within $(2N)^{1/2}$ is defined and odd prime number P_N within N is defined equally. In this way, the remainders of even number 2N divided by each P_i are given as well.

When 2N=6~8, there is only one prime number 2 within $(2N)^{1/2}$, i. e. module P_i of odd prime number more than 2 is not yet existent. When i = 0, there is one P_N prime number 3 within N, i. e.
$\pi(P_N)=1$.Obviously, this prime number P_N don't to sieving by sieve method of congruence classification. It is prime number P_b, whose number is $\pi(P_b)=1$.

When 2N=10~24, there is only one P_i prime number 3 within $(2N)^{1/2}$, P_n (the maximal prime number P_i) is 3; prime number P_N within N begins to have 3 and 5, then increases to have 3, 5and 7, at last increases to have 3, 5,7 and 11;When we taking 3 as module, prime number P_N can be divided into two congruence classes at least; Even if there is one congruence class congruent to even number 2N, there is still at least one congruence class not congruent to even number 2N after being sieved, thus prime umber P_b exists.

When 2N =26~34, there are two P_i Prime numbers 3 and 5 within $(2N)^{1/2}$, $P_n=5$; Prime number P_N within N begins to have 3, 5, 7, 11and 13, at last increases 3, 5, 7, 11, 13and 17. At first, When we taking P_n prime number 5 as module, prime number P_N can be divided into four congruence class; Even if there is one congruence class congruent to even number 2N, there are still three congruence class not congruent to even number 2N after being sieved. Then, when We taking P_i prime number 3 as module, the congruence classification of prime number P_N within three congruence class (which are taking 5 as module but being not congruent to even number 2N) can be divided into two congruence class at least; Also, even if there is one congruence class congruent to even number 2N, there is still at least one congruence class not congruent to even number 2N after being sieved. The prime number P_N within this congruence class (which not congruent to even number 2N) is prime number P_b.

When $2N\geqslant36$, $N\geqslant18$, and $(2N)^{1/2}\geqslant6$, i. e. $2(2N)^{1/2}\geqslant12$, then $N>(2N)^{1/2}$, the number of P_i prime number $\pi(P_i)\geqslant2$. According to Theorem3, there are two prime numbers P_{i+1} and P_{i+2} at least between $(2N)^{1/2}\sim2(2N)^{1/2}$, i.e. There are two prime numbers P_{i+1} and P_{i+2} at least between $(2N)^{1/2}\sim N$ i. e. the number of odd prime number within N $\pi(P_N)\geqslant i+2$. Thus, according to Theorem 4, using the sieve method of congruence classification and the inductive method, we have:

1.When i=3, i. e. there are three P_i prime numbers 3, 5, 7 within $(2N)^{1/2}$, 2N =50~120, and there are five P_N Prime numbers at least 3, 5, 7, 11, 13.

At first, when we taking 7 as module, prime number P_N can be divided into five congruence class at least; Even if there is one congruence class congruent to even number

2N, there are still at least four congruence class not congruent to even number 2N after being sieved.

Then, when we taking 5 as module, the congruence classification of prime number P_N within four congruence class (which are taking 7 as module but being not congruent to even number 2N) can be divided into three congruence class at least; also, even if there is one congruence class congruent to even number 2N, there are still at least two congruence class not congruent to even number 2N after being sieved.

At last, when we taking 3 as module, the congruence classification of prime number P_N within two congruence class (which are taking 5 module but being not congruent to even number 2N) can be divided into two congruence class at least; Also, even if there is one congruence class congruent to even number 2N, there is still at least one congruence class not congruent to even number 2N after being sieved, thus prime number P_b exists.

2. Given $i = K$, $K>3$, i. e. there are K prime numbers P_i within $(2N)^{1/2}$, the maximal prime number P_i is P_k, and taking P_k as module, prime number P_N can be divided into at least K+1 congruence classes; Even if there is one congruence class congruent to even number 2N, there are still at least K congruence classes not congruent to even number 2N after being sieved; Then continuously taking P_{k-1}, $P_{k-2}\cdots\cdots P_2$, P_1 as module in order respectively, the congruence classification of prime number P_N within the congruence class (which are taking previous one prime number of P_i as module but being not congruent to even number 2N) is conducted; also, even if there is always one congruence class congruent to even number 2N each time, through gradual sieving, at last there is still at least one congruence class not congruent to even number 2N, thus prime number P_b exists.

When $i= K+1$, i. e. there are K+1 prime number P_i within $(2N)^{1/2}$, the maximal prime number P_i is P_{k+1}, and taking P_{k+1} as module, prime number P_N can be divided into at least K+2 congruence classes; Even if there is one congruence class congruent to even number 2N, there are still K+1 congruence classes not congruent to even number 2N after being sieved. When taking P_k as module, P_1,$P_2\cdots\cdots P_{k-2}$, P_{k-1} are the remainder number of odd prime number, remainder number of P_k is zero, even remainder number r_1 of $P_{k+1} <$ even remainder number r_2 of P_{k+2}. Therefore, no matter any congruence class being sieved, which is in full conformity to the inductive assumption, the congruence classification of prime number P_N within K+1 congruence class(which are taking P_{k+1} module but being not congruent to even number 2N) can be divided into K+1 congruence class; Also, even if there is one congruence class congruent even number 2N, there are still K congruence class not congruent to even number 2N after being sieved; Then continuously taking P_{k-1} ,$P_{k-2}\cdots\cdots P_2$, P_1 as module in order respectively, the congruence classification of prime number P_N within congruence class (which are taking previous one prime number of P_i as module but being not congruent to even number 2N) is conducted; Also, even if there is always one congruence class congruent to even number 2N each time, through gradual sieving, at last there is still at least one congruence class not congruent to even number 2N, thus prime number P_b exists.

As stated above, when $2N > 4$, taking P_i as module, the prime number P_b exists always, Which being not congruent to even number 2N. Theorem 5 has proved.

In fact, P_n enlarges gradually and $\pi(P_N)$ increases continuously with an enlargement of even number 2N; Meanwhile, $\pi(P_N)$ increases with an enlargement of N, i. e. It increases with an enlargement of 2N, but $\pi(P_i)$ increases only with an enlargement of $(2N)^{1/2}$, thus an increase in $\pi(P_N)$ is more than that in $\pi(P_i)$. The greater even number 2N, the more an increase in $\pi(P_N)$ than that in $\pi(P_i)$. Therefore, taking P_i as module, the congruence classes of prime number P_N for a greater even number are more than i+1 but tend to P_i. Example 1 will show that 50 can represent greater even numbers. The congruence classes of prime number P_N are complete essentially. There is only one congruence classes miss and prime number P_b is more in the whole process of congruence class sieve. When even number 2N is great adequately, prime number P_N can be divided into P_i congruence classes, excepting that prime number P_N as a module P_i is taken away in the previous sieve and makes congruence class 0 miss. Example 2 will show that 100 can represent adequately great even numbers, prime number P_N can be completely divided into P_i congruence classes in the whole process of congruence class sieve, and more prime numbers P_b will appear. The systemic and many calculation has shown that there is only one prime numbers P_b for 6,8,12 in even numbers more than 4, while other even numbers have two or more prime numbers P_b. With an enlargement of even number 2N, the lower limit of $\pi(P_b)$ increases gradually on the basis of 1~2~3, and the upper limit of $\pi(P_b)$ increases continuously on the basis of 1~3~5~12~13. The greater even number 2N, the more prime number P_b. The proof of Theorem 5 has been finished.

According to Theorem 5, because P_b is prime number P_N taking P_i as module but not congruent to even number 2N, i. e. $2N \equiv P_b \pmod{P_i}$ is untenable, and that is to say P_i can' t divide exactly $2N-P_b$, thus $2N-P_b$ is prime number. Given $2N-P_b=q$ (q is prime number), then even number 2N more than 4 have the general solution of "1+1":

$$2N = P_b + q$$

i.e.

$$2N = P_b + (2N - P_b) \quad \ldots\ldots\ldots\ldots(1)$$

Ex.1: Extract "1+1"solution of 50.

Analysis: The mean number of 50 is 25, P_i is 3, 5 and 7, and prime number P_N has 3, 5, 7, 11, 13, 17, 19 and 23 etc.

Step 1: Extract prime number P_b:

(1) Taking 7 as module, prime number P_N can be divided into six congruence classes, which are just congruence class 1 without prime number P_N, and have not prime number class of P_N congruent to 50, thus it is not necessary to sieve them.

(2) Taking 5 as module, prime number P_N in six congruence classes taking 7 as module but not congruent to 50 are divided into five complete congruence classes, and 5 of prime number P_N of congruence class 0 congruent to 50 is sieved.

(3) Taking 3 as module, prime number P_N in four congruence classes taking 5 as module but not congruent to 50 are divided into three complete congruence classes, and

11,17, 23 of prime number P_N of congruence classes 2 congruent to 50 are sieved. The retained prime number P_N of two congruence classes is prime number P_b, which has 3, 7, 13, and 19.

Step 2: Extract "1+1" solution:

Substituting 50 into 2N in Equation (1), and substituting prime number P_b obtained from 50 into P_b in Equation (1) respectively, we can obtain "1+1" solution of 50:50=3+47, 7+43, 13+37, 19+31.

Taking P_i as module, the congruence class for prime number P_N of even number 50 approaches to being complete, and only one congruence class misses in the whole process of congruence class sieve, with more "1+1" solutions, which can represent generally greater even number.

Ex. 2: Extract "1+1"solution of 100:

Analysis: The mean number of 100 is 50, P_i is 3, 5 and 7, prime number P_N has 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43and 47 etc.

Step 1: Extract prime number P_b:

(1) Taking 7 as module, prime number P_N is divided into seven complete congruence classes, and 23 and 37 of prime number P_N of congruence class 2 congruent to 100 are sieved.

(2) Taking 5 as module, prime number P_N in six congruence classes taking 7 as module but not congruent to 100 is divided into five complete congruence classes; And 5 of prime number P_N of congruence class 0 congruent to 100 is sieved.

(3) Taking 3 as module, prime number P_N in four congruence classes taking 5 as module but not congruent to 100 is divided into three complete congruence classes, and 7, 13, 19, 31 and 43 of prime number P_N of congruence class 1 congruent to 100 are sieved. The retained prime number P_N of two congruence classes is prime number P_b, which has 3, 11, 17, 29, 41and 47 etc.

Step 2:Extract "1+1" solution:

Substituting 100 in to 2N in Equation (1), and substituting prime number P_b obtained from 100 into P_b in Equation (1) respectively, we can obtain "1+1" solution of 100:100=3+97, 11+89, 17+83, 29+71, 41+59, 47+53.

Taking P_i as module, prime number P_N of even number 100, in the whole process of congruence class sieve, can be divided into P_i complete congruence classes, with six "1+1"solutions, which can represent adequately greater even number.

As stated above, 4 has the specific "1+1" solution, and even number more than 4 has general "1+1"solution. The greater even number, the more "1+1" solutions tend to be.

Proposition (1) has been proved.

3 THE PROOF OF PROPOSITON (2)

Proposition (2): Each odd number more than 5 is the sum of three prime numbers.

Demonstration: In fact, Proposition (2) is the inference of Proposition (1), thus

Proposition (1) has been proved, then Proposition (2) is easy to be proved also. Generally, given odd number M more than 5, S is prime number, M-2 >S>2, then M-S is the even number more than 2.

If M-S =4, then M-S =2+2,

i.e. $M=2+2+S$............................(2—1)

If M-S is even number 2N more than 4, according to Equation (1), we have

$$M\text{-}S = P_b + (2N\text{-} P_b)$$

i.e. $M= P_b + (2N\text{-} P_b)+S$..................(2---2)

Also, when M>11, Equation (2—2) has multiple solutions.

According to Equation (2---1) and (2---2), when M>7, Proposition (2) has multiple solutions. The greater M, the more solutions.

Ex. 3:Extract that 15 is the solution for the sum of three prime numbers

Analysis: Suppose S is prime number, according to the defined condition of S: 15-2 >S>2, S has four prime numbers, i. e. 3, 5, 7 and 11; according to Equation (2---1) and (2—2), taking away the same solutions, we have:

$$15 = 7 + 5 + 3,\ 5+5+5,\ 2 + 2 +11.$$

As stated above, Proposition (2) has been proved.

4 CONCLUSIONS

The proof of Proposition (1):

"1+1" solution for even number 4: 4= 2+2.

This method is called as the sieve method of congruence classification, which the definite prime number is taken as module, and the definite type numbers are divided into congruence class, then the congruence class congruent to module 0 of the definite prime number is sieved. According to the following five theorems, the sieve method of congruence classification can be used to prove that each even number more than 4 has the general "1+1" solution.

Theorem 1: If P_c is prime number more than 3, the continuous numbers within P_c are taken optionally from 6a-5 type or 6b-7 type number progression, then only one number at most in these continuous numbers can render that $6a\equiv5 (\mathrm{mod}\ P_c)$ or $6b\equiv7(\mathrm{mod}\ P_c)$ is tenable, but all other numbers can not render that $6a\equiv5(\mathrm{mod}\ P_c)$ or $6b\equiv7(\mathrm{mod}\ P_c)$ are tenable.

Theorem 2: If P_A is the definite prime number more than 5, two prime numbers at least exist within P_A~$2P_A$.

Also, Theorem 2 can be formulated as Theorem 3 equivalent to it.

Theorem 3: Given integer B>5, there are two prime numbers at least between B~2B.

Theorem 4: If even number more than 4 is 2N, odd prime number P_N within N, taking an optional odd prime number P_i within $(2N)^{1/2}$ as module, can be divided into i+1 congruence classes at least.

Theorem5: Given an optional even number 2N more than 4, odd prime number

within $(2N)^{1/2}$ is P_i, and odd prime number within mean number N of this even number is P_N, then P_b of prime number P_N taking P_i as module but not congruent to even number 2N exists inevitably.

According to Theorem 5, even number 2N more than 4 has the general "1+1" solution: $2N=P_b+(2N-P_b)$.

Synthesis 4 and even numbers more than 4 have "1+1" solution. Proposition (1) has been proved.

The proof of Proposition (2): Suppose odd number M is more than 5, S is prime number, $M-2>S>2$, then M-S is even number more than 2.

If $M-S=4$, then $M-S=2+2$,

i.e.

$$M=2+2+S$$

If M-S is even number 2N more than 4, according to Equation (1), we have

$$M-S=P_b+(2N-P_b)$$

i.e.

$$M=P_b+(2N-P_b)+S$$

Proposition (2) has been proved.

Both Proposition (1) and (2) have been proved generally, thus Goldbach conjecture has been proved generally completely. For the future, Goldbach conjecture should be renamed as Goldbach theorem.

Notes:

① Proposition (1) can be called as Proposition "1+1"also. In order to simplify it, "1+1" is cited in this paper.

② Including mean number of even number. Ditto in this paper.

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哥德巴赫猜想的分析—归纳法证明*

提要：本文运用分析—归纳法，系统地对大于 2 的偶数的（1+1）解进行了充分的分析，归纳出四条具体的特殊规律和一条总体的一般规律，一般地证明了哥德巴赫猜想完全成立。

关键词：哥德巴赫猜想，分析—归纳法，四条特殊规律，一条一般规律，一般证明，完全成立。

一、序言

哥德巴赫猜想是数论的著名难题之一。由于某位数学家著作的影响，哥德巴赫猜想在我国长期被不完整不准确地理解。该位数学家在著作中“提示：凡大于 4 之偶数必为二奇素数之和，此乃著名的 Goldbach（哥德巴赫）问题。”因此，他的门生把他的话当成标准的定义，把“任何≥6 的偶数都是两个奇素数之和”叫做哥德巴赫猜想[1]。这是既不完整也不准确的。该位数学家紧接着上面那句话还有两句很特别的话：“若此定理（未得到证明怎么就能称为定理而作为论据？这是“窃取论点”的逻辑错误。）真实，则吾人可以证明：凡大于 7 之奇数必为三个奇素数之和。因若 n 为大于 7 之奇数，则 n-3 乃大于 4 的偶数，故 $n-3=p_1+p_2$，即 $n=3+p_1+p_2$。”由于该位数学家后面这两句话，虽然它们显然是在一个大假设的前提下、借“吾人”的名义被提出来的，他的门生在著作中有时又给哥德巴赫猜想增加一个内容：“任何≥9 的奇数都是三个奇素数之和。”[3]这样，哥德巴赫猜想的内容完整了，但还是不准确的。哥德巴赫于 1742 年（当时由于 1 也规定为素数）通过大量的试算，提出两个命

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题：（1）每个偶数都是两个素数之和；（2）每个大于1的奇数都是三个素数之和。他由于不能证明，称两个命题为猜想。他写信给当时的数学家欧勒（L.Euler），请他对这个猜想从理论上给予证明。欧勒认为这个猜想是真实的，但也无法做出证明。从此，这个猜想便成为数论最著名的难题之一。后来，国际数学家大会为了维持因式分解定理的唯一性，决定1不是素数也不是合数，因而哥德巴赫猜想的两个命题应该相应改成下列形式：（1）每个大于2的偶数都是两个素数之和；（2）每个大于5的奇数都是三个素数之和。两个命题统称为哥德巴赫猜想。第一个命题又叫做偶数哥德巴赫猜想，简称为（1+1）；第二个命题又称为奇数哥德巴赫猜想，简称为（1+1+1）。据报道说，有的人对一个一个的偶数进行了验算，一直到4×10^{14}的数，都说明命题（1）成立。但是，要对命题（1）从理论上进行一般的证明，直到二十世纪初期还无从下手。到了二十世纪二十年代以后，数学家们才开辟出两条探索的路线：1920年挪威数学家布伦（Brum）运用筛法，得出了每一个大偶数都是两个“素因子都不超过九个的”数之和的结果，开辟了（9+9）~（7+7）~（6+6）~（5+5）~（4+4）~（3+3）~（2+3）的探索路线；1948年匈牙利数学家兰恩易（A.Renyi）运用指数和的方法，证明了每一个大偶数都是一个素数和一个“素因子不超过六个的”数之和，开辟了（1+6）~（1+5）~（1+4）~（1+3）~（1+2）的探索路线。两条探索路线的特点是相同的：（1）两条探索路线都采用间接证明的方法；（2）两条探索路线都是以大偶数为前提的，认为大偶数的问题解决了，不管大偶数之前有多少非大偶数都是可以验证的，哪怕证明大偶数的结果不符合非大偶数也无关紧要；（3）两条探索路线的两个或一个加数都是合数，都要求在间接证明的过程中，把这两个或一个合数变成只有一个素数因子的素数。几十年来，我国和外国的许多数学家在两条探索路线上耗费了大量的时间和精力，可是根本解决不了问题。两条探索路线采用间接证明的方法是可以的，正如登山可以走迂回的路线一样，只要证明人不怕浪费时间和精力。不过它们毕竟都只是充满着失败可能性的探索路线，而决不是肯定可以达到目的的正确道路。它们以大偶数为前提也无可非议，虽然大偶数就是想象的充分大的偶数，是一个抽象的偶数概念。它没有确定的界限，比10的几十万次方还要大，比任何一个特定的偶数都要大，要多大就有多大。两条探索路线采取由一般到特殊、由抽象到具体、由大到小的论证，也可以理解为一种论证方法，虽然它们的做法与人类生活经验背道而驰。但是，在两条路线上探索的人认为大偶数的问题解决了，而不管大偶数之前有多少非大偶数都是可以验证的，那就不一定了，因为大偶数之前实际上存在着一个无穷的偶数数列。他们认为证明大偶数的结果不符合非大偶数也无关紧要，那就是根本错误的。例如（1+2）对于大偶数成立，但显然对于非大偶数4和10就不成立，不过4和10的（1+1）解很容易分别求得；那么（1+2）对于一个特定的非大偶数2^{50000}是否成立？它的（1+2）解是什么？它的（1+1）解又是什么？至于两条探索路线都要求在间接证明的过程中，把两个或一个合数变成一个素数是根本不可能的。我们知道，合数就是有两个或两个以上素数因子的数，而素数就是只有一个素数因子的数。因此，（9+9）证明到（2+3），甚至（2+3）再证明到（2+2），或者（1+6）证明到（1+2），都还没有改变两个或一个合数的性质，只是在形式上减少一个或几个素数因子而已。但是，（2+2）或（1+2）要证明到（1+1）时，两个或一个合数就要改变成素数了。这就是说，假如以（1+6）为起点的探索路线的证明行得通，则有下列关系式：

一个大偶数=（1+6）=（1+5）=（1+4）=（1+3）=（1+2）= （1+1）
每个数用不名数的数字表示，这是一个数字计算式，看不出会有什么问题。每个数按照原义用名数表示，前面五个等式也看不出会有什么问题；但最后一个等式便变成了一个数理逻辑悖论等式：

一个奇素数+一个有两个奇素数因子的合数= 一个奇素数+一个奇素数

为了突出上式各个数的本质关系，把等号两边各个数的修饰语去掉，得

素数+合数=素数+素数

假如以（9+9）为起点的探索路线的证明也行得通，结果也是一样，只是悖论更加突出：

合数+合数=素数+素数

两式都表明，合数就是素数，素数也就是合数，合数与素数就没有什么区别了。我们知道，自然数除了 1 既不是素数也不是合数（也可以说 1 既是素数也是合数）以外，不是素数，就是合数。这正如人除了两性人以外，不是男人，就是女人。如果两性人比喻 1，男人比喻素数，女人比喻合数，上面二式便变成

男人+女人=男人+男人

以及　　女人+女人=男人+男人

这样一来，女人就是男人，男人也就是女人，女人与男人就没有什么区别了。这样的数理逻辑错误对小学生来说也是一目了然的。这个悖论还要用高深的数学理论去证明，人类不是在开自己的玩笑吗？（1+2）的研究成果并不是像有的人所说的那样距离（1+1）只有一步之遥了，而是说明以（1+6）为起点的探索路线已经走到了绝路的尽头，不能再前进一步了。以（9+9）为起点的探索路线还没有走到绝路的尽头，还有一步（2+2）可走，如果有人硬要去浪费生命的话。

哥德巴赫猜想提出的是一般形式的问题，证明它就必须把握>2 的偶数的（1+1）解的发展规律。我们证明哥德巴赫猜想时，不能运用以前的老方法，而必须创造出新的证明方法。本文采用分析——归纳法（也可以叫做趋势分析法），系统地对大于 2 的偶数的（1+1）解进行了充分的分析，归纳出四条具体的特殊规律，最后综合出一条总体的一般规律，从而一般地证明了哥德巴赫猜想完全成立。

二、偶数哥德巴赫猜想的证明

偶数哥德巴赫猜想：每个大于 2 的偶数都是两个素数之和。

为了把握大于 2 的偶数的（1+1）解的发展规律，我们从已知到未知、从小到大、从具体到一般地，对大于 2 的偶数的（1+1）解进行下列系统的考察。为了便于考察和分析，我们设 2N 为大于 2 的偶数，（1+1）为 2N 偶数的（1+1）解，π（1+1）为 2N 偶数的（1+1）解的个数。

（一）2N=4～100 的（1+1）解的详细考察，结果见表 1。

（二）2N=100～1000 的（1+1）解的一般考察，结果见表 2。

（三）2N=1000～5000 的（1+1）解的一般考察，结果见表 3。

从表 1 至表 3 的资料我们可以看出，大于 2 的偶数的（1+1）解有下列四条具体的

特殊规律：

1、偶素数 2 只能构成偶数 4 的（1+1）解：4=2+2。这是唯一的。偶素数 2 不能构成>4 的偶数的（1+1）解。>4 的偶数的（1+1）解都只能由奇素数构成。

2、设 N 等于素数 P，则 2N 偶数即为 2P 型偶数。任何素数 P 都可以构成 2P 型偶数。所有 2P 型偶数都有一个相等素数的（1+1）解：2N=2P=P+P。2P 型偶数 4 和 6 都只有一个相等素数的（1+1）解。>6 的 2P 型偶数，除了一个相等素数的（1+1）解以外，还有一个或一个以上的不相等素数的（1+1）解。

3、设 P_1 和 P_2 为两个不相等的奇素数，且 $P_2>P_1$，α 为一个正整数，则 P_1 和 P_2 在>6 的偶数的不相等素数的（1+1）解 $2N=P_1+P_2$ 中，有下列关系：

$$P_1=N-\alpha$$
$$P_2=N+\alpha$$

P_1 和 P_2 以 2N 偶数的中数 N 为中心相对称，α 为对称距。如果 N 为偶数，则 α 为奇数；如果 N 为奇数，则 α 为偶数。因此，P_1 与 P_2 又有“偶心奇对称”和“奇心偶对称”的规律。构成>6 的偶数的不相等素数的（1+1）解的两个素数 P_1 和 P_2，都是以 N 为对称中心的素数对。运用对称素数对的存在，我们就可以运用解二元一次不定方程的方法，求解>6 的 2N 偶数的不相等素数的（1+1）解。有多少个以 N 为对称中心的素数对存在，2N 偶数就有多少个不相等素数的（1+1）解。>6 的一定偶数的不相等素数的（1+1）解的求解步骤如下：

（1） 把 2N 偶数之内的奇素数列出；

（2） 把[2，2N]区间分为[2，N]区间和[N，2N]区间；

（3） 把[2，N]区间内的奇素数代入 $P_2=2N-P_1$ 式的 P_1，看 P_2 是否存在。P_2 如果存在，则与 P_1 组成一个对称素数对，2N 得到一个（1+1）解；P_2 如果不存在，则 P_1 不能组成对称素数对，舍去；如此继续进行下去，直至把全部 P_1-P_2 对称素数对求出。或者反过来，把[N，2N]区间的素数代入 $P_1=2N-P_2$ 式的 P_2，看 P_1 是否存在，找出全部 P_1-P_2 对称素数对，从而求出 2N 偶数的不相等素数的全部（1+1）解。实际上 2P 型偶数的相等素数的（1+1）解，只是 2N 偶数的不相等素数的（1+1）解的特殊形式（$P_1=P_2$，α=0），可以统一求出。

4、对于大于 2 的偶数，仅仅 4、6、8 和 12 四个偶数，它们每个只有一个（1+1）解，其它偶数都有两个或两个以上的（1+1）解。（1+1）解的个数，在 2N=4～100 时为 1～9 个，在 2N=100～1000 时为 6～48 个，在 2N=1000～5000 时为 30～104 个，局部虽然在一定范围内有不规则的变化，但总的发展趋势是：随着 2N 偶数的逐渐增大，2N 偶数的（1+1）解的个数迅速增加。因此，只要大于 2 的小偶数（如≤12）有（1+1）解，较大的偶数（如≥14）都至少有两个或两个以上的（1+1）解，而且随着偶数的增大，偶数的（1+1）解的个数总的发展趋势是不断增多。

综合上述 2N 偶数的（1+1）解的四条具体的特殊规律，我们可以得出一条总体的一般规律：每个大于 2 的偶数至少有一个（1+1）解。因此，偶数哥德巴赫猜想得到了证明：

$$2N=P_1+P_2,\ P_1\leq P_2.$$

三、奇数哥德巴赫猜想的证明

奇数哥德巴赫猜想 ：每个大于 5 的奇数都是三个素数之和。

证明：奇数哥德巴赫猜想实际上是偶数哥德巴赫猜想的推论，所以后者得到了证明，则前者容易证明：设奇数 M 大于 5，q 为素数，M-2>q>2，则 M-q 为大于 2 的偶数。因此，我们有

$$M-q=P_1+P_2,\ P_1\leqslant P_2,$$

即

$$M=P_1+P_2+q.$$

当 M 大于 7 时，M 都有两个或两个以上的（1+1+1）解。M 越大，解越多。奇数哥德巴赫猜想得到了证明。

四、结论

本文系统地从已知到未知、从小到大、从具体到一般地，对大于 2 的偶数的（1+1）解进行了充分的分析，归纳出 2N 偶数的（1+1）解的四条具体的特殊规律，最后综合出一条总体的一般规律：每个大于 2 的偶数至少有一个（1+1）解。因此，我们一般地证明了“每个大于 2 的偶数都是两个素数之和”：

$$2N=P_1+P_2,\ P_1\leqslant P_2.$$

根据偶数哥德巴赫猜想的证明，“每个大于 5 的奇数都是三个素数之和”也得到了证明：设 M 为大于 5 的奇数，q 为素数，M-2>q>2，则 M-q 为大于 2 的偶数。因此，我们有

$$M-q=P_1+P_2,\ P_1\leqslant P_2$$

即

$$M=P_1+P_2+q$$

作者一般地证明了哥德巴赫猜想完全成立。

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4~100的（1+1）解记录表

表1

2N	（1+1）	π（1+1）
4	2+2.	1
6	3+3.	1
8	3+5.	1
10	3+7，5+5.	2
12	5+7.	1
14	3+11，7+7.	2
16	3+13，5+11.	2
18	5+13，7+11.	2
20	3+17，7+13.	2
22	3+19，5+17，11+11.	3
24	5+19，7+17，11+13.	3
26	3+23，7+19，13+13.	3
28	5+23，11+17.	2
30	7+23，11+19，13+17.	3
32	3+29，13+19.	2
34	3+31，5+29，11+23，17+17.	4
36	5+31，7+29，13+23，17+19.	4
38	7+31，19+19.	2
40	3+37，11+29，17+23.	3
42	5+37，11+31，13+29，19+23	4
44	3+41，7+37，13+31	3
46	3+43，5+41，17+29，23+23	4
48	5+43，7+41，11+37，17+31，19+29	5
50	3+47，7+43，13+37，19+31	4
52	5+47，11+41，23+29	3
54	7+47，11+43，13+41，17+37，23+31	5
56	3+53，13+43，19+37	3
58	5+53，11+47，17+41，29+29	4
60	7+53，13+47，17+43，19+41，23+37，29+31	6
62	3+59，19+43，31+31	3
64	3+61，5+59，11+53，17+47，23+41	5
66	5+61，7+59，13+53，19+47，23+43，29+37	6
68	7+61，31+37	2
70	3+67，11+59，17+53，23+47，29+41	5

2N	(1+1)	π (1+1)
72	5+67，11+61，13+59，19+53，29+43，31+41	6
74	3+71，7+67，13+61，31+43，37+37	5
76	3+73，5+71，17+59，23+53，29+47	5
78	5+73，7+71，11+67，17+61，19+59，31+47，37+41	7
80	7+73，13+67，19+61，37+43	4
82	3+79，11+71，23+59，29+53，41+41	5
84	5+79，11+73，13+71，17+67，23+61，31+53，37+47，41+43.	8
86	3+83，7+79，13+73，19+67，43+43.	5
88	5+83，17+71，29+59，41+47.	4
90	7+83，11+79，17+73，19+71，23+67，29+61，31+59，37+53，43+47.	9
92	3+89，13+79，19+73，31+61.	4
94	5+89，11+83，23+71，41+53，47+47.	5
96	7+89，13+83，17+79，23+73，29+67，37+59，43+53.	7
98	19+79，31+67，37+61.	3
100	3+97，11+89，17+83，29+71，41+59，47+53.	6

100～1000 的（1+1）解记录表

表 2

2N	(1+1)	π (1+1)
100	3+97, 11+89, 17+83, 29+71, 41+59, 47+53.	6
200	3+197, 7+193, 19+181, 37+163, 43+157, 61+139, 73+127, 97+103.	8
300	7+293, 17+283, 19+281, 23+277, 29+271, 31+269, 37+263, 43+257, 59+241, 61+239, 67+233, 71+229, 73+227, 89+211, 101+199, 103+197, 107+193, 109+191, 127+173, 137+163, 149+151.	21
400	3+397, 11+389, 17+383, 27+373, 41+359, 47+353, 53+347, 83+317, 89+311, 107+293, 131+269, 137+263, 149+251, 167+233, 173+227.	15
500	13+487,37+463,43+457,61+439,67+433,79+421, 103+397, 127+373, 151+349, 163+337, 193+307, 223+277, 229+271.	13
600	7+593, 13+587, 23+577, 29+571, 31+569, 37+563, 43+557, 53+547, 59+541, 79+521, 97+503, 101+499, 109+491, 113+487, 137+463, 139+461, 151+449, 157+443, 167+433, 179+421, 181+419, 191+409, 199+401, 211+389, 227+373, 233+367, 241+359, 251+349, 263+337, 269+331, 283+317, 293+307.	32
700	17+683, 23+677, 41+659, 47+653, 53+647, 59+641, 83+617, 101+599, 107+593, 113+587, 131+569, 137+563, 179+521, 191+509, 197+503, 233+467, 239+461, 251+449, 257+443, 269+431, 281+419, 311+389, 317+383, 347+353.	24
800	3+797, 13+787, 31+769, 43+757, 61+739, 67+733, 73+727, 109+691, 127+673, 139+661, 157+643, 181+619, 193+607, 199+601, 223+577, 229+571, 277+523, 313+487, 337+463, 367+433, 379+421.	21
900	13+887, 17+883, 19+881, 23+877, 37+863, 41+859, 43+857, 47+853, 61+839, 71+829, 73+827, 79+821, 89+811, 103+797, 113+787, 127+773, 131+769, 139+761, 149+751, 157+743, 167+733, 173+727, 181+719, 191+709, 199+701, 223+677, 227+673, 239+661, 241+659, 257+643, 269+631, 281+619, 283+617, 293+607, 307+593, 313+587, 331+569, 337+563, 353+547, 359+541, 379+521, 397+503,	48

	401+499，409+491，421+479，433+467，439+461，443+457.	
1000	3+997，17+983，23+977，29+971，47+953，53+947，59+941，71+929，89+911，113+887，137+863，173+827，179+821，191+809，227+773，239+761，257+743，281+719，317+683，347+653，353+647，359+641，353+647，359+641，383+617，401+599，431+569，443+557，479+521，491+509.	30

1000～5000 的（1+1）解记录表

表 3

2N	（1+1）	π（1+1）
1000	3+997，17+983，23+977，29+971，47+953，53+947，59+941，71+929，89+911，113+887，137+863，173+827，179+821，191+809，227+773，239+761，257+743，281+719，317+683，347+653，353+647，359+641，353+647，359+641，383+617，401+599，431+569，443+557，479+521，491+509.	30
2000	3+1997，7+1993，13+1987，67+1933，127+1873，139+1861，199+1801，211+1789，223+1777，241+1759，277+1723，307+1693，331+1669，337+1663，373+1627，379+1621，421+1579，433+1567，457+1543，541+1459，547+1453，571+1429，577+1423，601+1399，619+1381，673+1327，709+1291，751+1249，769+1231，787+1213，829+1171，877+1123，883+1117，907+1093，937+1063，967+1033，991+1009.	37

2N	(1+1)	π (1+1)
3000	29+2971, 31+2969, 37+2963, 43+2957, 47+2953, 61+2939, 73+2927, 83+2917, 97+2903, 103+2897, 113+2887, 139+2861, 149+2851, 157+2843, 163+2837, 167+2833, 181+2819, 197+2803, 199+2801, 211+2789, 223+2777, 233+2767, 251+2749, 269+2731, 271+2729, 281+2719, 293+2707, 307+2693, 311+2689, 313+2687, 317+2683, 337+2663, 353+2647, 367+2633, 379+2621, 383+2617, 409+2591, 421+2579, 443+2557, 449+2551, 457+2543, 461+2539, 479+2521, 523+2477, 541+2459, 563+2437, 577+2423, 601+2399, 607+2393, 617+2383, 619+2381, 643+2357, 653+2347, 659+2341, 661+2339, 691+2309, 719+2281, 727+2273, 733+2267, 757+2243, 761+2239, 787+2213, 797+2203, 821+2179, 839+2161, 857+2143, 859+2141, 863+2137, 887+2113, 911+2089, 919+2081, 937+2063, 947+2053, 971+2029, 983+2017, 997+2003, 1013+1987, 1021+1979, 1049+1951, 1051+1949, 1069+1931, 1087+1913, 1093+1907, 1123+1877, 1129+1871, 1153+1847, 1213+1787, 1217+1783, 1223+1777, 1259+1741, 1277+1723, 1279+1721, 1291+1709, 1301+1699, 1303+1697, 1307+1693, 1373+1627, 1381+1619, 1399+1601, 1429+1571, 1433+1567, 1447+1553, 1451+1549, 1489+1511.	104

2N	(1+1)	π (1+1
4000	11+3989, 53+3947, 71+3929, 83+3917, 89+3911, 137+3863, 149+3851, 167+3833, 179+3821, 197+3803, 233+3767, 239+3761, 281+3719, 383+3617, 419+3581, 443 +3557, 461+3539, 467+3533, 509+3491, 587+3413, 593+3407, 641+3359, 653+3347, 677+3323, 701+3299, 743+3257, 797 +3203, 809+3191, 863+3137, 881+3119, 911+3089, 977+3023, 1031+2969, 1061+2939, 1091+2909, 1097+2903, 1103+2897, 1163+2837, 1181+2819, 1223+2777, 1259+2741, 1289+2711, 1301+2699, 1307+2693, 1367+2633, 1409+2591, 1451+2549, 1523+2477, 1553+2447, 1559+2441, 1583+2417, 1601+2399, 1607+2393, 1619+2381, 1667+2333, 1733+2267, 1787+2213, 1847+2153, 1871+2129, 1889+2111, 1901+2099, 1913+2087, 1931+2069, 1973+2027, 1997+2003.	65
5000	7+4993, 13+4987, 31+4969, 43+4957, 67+4933, 97+4903, 139+4861, 199+4801, 211+4789, 241+4759, 271+ 4729, 277+4723, 337+4663, 349+4651, 379+4621, 397+4603, 409+4591, 433+4567, 439+4561, 487+4513, 577+4423, 643 +4357, 661+4339, 673+4327, 727+4273, 739+4261, 757+4243, 769+4231, 823+4177, 907+4093, 997 +4003, 1033+3967, 1069+3931, 1093+3907, 1123+3877, 1153+3847, 1231+3769, 1291+3709, 1303+3697, 1327+3673, 1429+3571, 1453+3547, 1459+3541, 1471+3529, 1483+3517, 1489+3511, 1531+3469, 1543+3457, 1567+3433, 1609+3391, 1627+3373, 1657+3343, 1669+3331, 1693+3307, 1699+3301, 1741+3259, 1747+3253, 1783+3217, 1831+3169, 1879+3121, 1933+3067, 1951+3049, 1999+3001, 2029+2971, 2083+2917, 2113+2887, 2143+2857, 2203+2797, 2251+2749, 2269+2731, 2281+2719, 2287+2713, 2293+2707, 2311+2689, 2341+2659, 2383+2617.	76

The Proof Of Goldbach Conjecture
By Analysis — Induction Method ※

Abstract: This paper use the analysis — induction method to make the full analysis of (1+1) solution of >2 even number systematically, and induced four specific special rules and one general rule as whole. Goldbach conjecture has generally been proved to be tenable completely.

Key Words: Goldbach conjecture, Analysis — induction method, Four specific rules, One general rule, Prove generally, To be tenable completely.

1 Introduction

Goldbach conjecture is one of the well known problems in the theory of number. In China, Goldbach conjecture has been understood incompletely and inaccurately for the long time, due to the effects of a mathematician's works. He wrote: "What all even numbers greater than 4 must be the sum of two odd primes is the well known Goldbach problem". His students, therefore, took his word as the standard definition, and called "any ≥6 even number is the sum of two odd primes as Goldbach conjecture." [1] This

※This treatise has won the international excellent article prize. And it has published in network magazine "International Outstanding Paper " on March 2005, Net address: www.WorldThesis.Com. Its copyright was registered in China on September 26,2005, Approved Registration Number: 19—2005—A—0040.

expression is both incomplete and inaccurate. Following this word above, this mathematician said two special words that "if this theorem (How can it be called as theorem and used as grounds of argument on condition that it has not been proved? This is a logistic mistake by argument.) is true, I can prove that all odd numbers greater than 7 must be the sum of three odd primes; If n is odd number greater than 7, n-3 is still even number greater than 4, so $n-3 = P_1 + P_2$, i.e. $n = 3 + P_1 + P_2$." Though the two sentences of words this mathematician said were delivered obviously in the name of "I" under the prerequisite of a great assumption, his student, sometimes, added "any $\geqslant 9$ odd number is the sum of three odd primes" [3] to Goldbach conjecture again. In this way, the content of Goldbach conjecture is complete but inaccurate. In 1742 (1 was also defined to be prime number at that time), Goldbach presented two propositions based on many calculation: (1) Each even number is the sum of two prime numbers; (2) Each odd number greater than 1 is the sum of three prime numbers. Because he could not prove these two propositions, so they were called as conjecture. He wrote to L. Euler, who was mathematician at that time, and asked him to prove this conjecture theoretically. Euler considered that this conjecture was true, but he could not prove it also. Since that time, this conjecture has become one of the well known problems in the theory of number. In order to keep factorization theorem unique, the International Congress of

Mathematicians decided that 1 is not prime number and composite number, so two propositions of Goldbach conjecture should be changed in such a way: (1) Each even number greater than 2 is the sum of two prime numbers; (2) Each odd number greater than 5 is the sum of three prime number. These two propositions are called as Goldbach conjecture. The first proposition is called as Goldbach conjecture of even number, which is called (1 + 1) for short. The second proposition is called as Goldbach conjecture of odd number, which is called (1 + 1 + 1) for short. It was reported that someone did the checking computation of even numbers one by one, and continued to 4×10^{14}. All results demonstrated Proposition (1) to be tenable. However, up to the early 1900's, the general proof of Proposition (1) could be done theoretically. After 1920's, mathematicians opened two research routes. In 1920, Brum, mathematician in Norway, used the sieve method to obtain the result that each great even number is the sum of two numbers having "prime factors not more than 9 in number" and opened the research routes of (9 + 9) ~ (7 + 7) ~ (6 + 6) ~ (5 + 5) ~ (4 + 4) ~ (3 + 3) ~ (2 + 3). In 1948, A. Renyi, mathematician in Hungary, used the exponential sum method to prove that each great even number is the sum of one prime number and one number having "prime factors not more than 6 in number" and opened the research routes of (1 + 6) ~ (1 + 5) ~ (1 + 4) ~ (1 + 3) ~ (1 + 2). The features of these two research routes are same. (1) The indirect proof method was used for

these two research routes. (2) These two research routes took great even number as prerequisite. It was considered that if only great even number problem was solved, all non-great even numbers in the front of great even number can be checked, furthermore, it was not important that the result of proving great even number was not conformable to non-great even number also. (3) The two research routes was that two or one addend is composite number, and it was required that the two or one composite number was turned to the prime number having only one prime factor in the process of the indirect proof. In the past several decade years, many mathematicians at home and abroad expended much time and great efforts in the two research routes, but this problem was not solved at all. The method that indirect proof was used by the two research routes is feasible. This is like that climbing can take the detour line, if only the person who proves this problem dares to expend time and efforts. However, these were a research route with fail possibility after all and it will never be sure that these were a correct research route attaining the goal. Also, what these research routes took great even number as prerequisite is blameless. Though great even number is an imaginative full great even number, great even number is an abstract concept of even number. Such even number has not the determined limit, greater than several hundred thousand power of 10, and greater than any specific even number, i.e. it can be greater as request. These two research routes used

a method of proof from general to special, from abstract to specific and from big to small. This doing was contrary to the life experience of human. However, the persons who made an approach on the two research routes considered that the problem on great even number was solved. It is not sure that great even number can be checked regardless of that how many non-great even numbers exist in the front of great even number, because an infinite sequence of even numbers exist in the front of great even number. They believed that no importance to that the result of proving great even number was not conformable to non-great even number. This is an essential mistake. For example: (1 + 2) is tenable under the condition of great even number, but is not tenable under the condition of 4 and 10 (non-great even number) obviously. However, it is easy to derive (1 + 1) solution of 4 and 10 respectively. Based on this, is (1 + 2) tenable under the condition of one specific non-great even number (2^{50000})? What is its (1 + 2) solution? What is its (1 + 1) solution? In the process of indirect proof, the two research routes required that two or one composite number is turned to one prime number. This is impossible at all. We know that composite number is a number having two or more than two prime factors, but prime number is number having only one prime number factor. Therefore, all proof from (9 +9) to (2 + 3), even from (2 + 3) to (2 + 2), or from (1 + 6) to (1 + 2) have not changed property of two or one composite number. That is all one or several

prime factors are reduced only in the form. However, when proof of (2 + 2) or (1 + 2) enters into (1 + 1), two or one composite number will be changed into prime number, that is to say, if the proof of the research route taking (1 + 6) as starting point is feasible, we have a relational expression as follow:

One Great Even Number = (1 + 6) = (1 + 5) = (1 + 4) = (1 + 3) = (1 + 2) = (1 + 1)

Every number is expressed by digit of abstract number. This is a formula of digital calculation, so no problem can be seen. Every number is expressed by concrete number according to original meaning, but no problem can be seen from front five equalities. However, the last one equality becomes one not conformable to mathematical logic:

One Odd Prime Number + One Composite Number Having Two Odd Prime Number Factors

= One Odd Prime Number + One Odd Prime Number

In order to highlight the essential relation of every number in the equality above, if the modifier of every number on two sides of equal mark is discarded, we have:

Prime Number + Composite Number = Prime Number + Prime Number

If the proof of the research route taking (9 + 9) as starting point is feasible and the result is same also, absurdity is more obvious only:

Composite Number + Composite Number = Prime Number + Prime Number

These two equalities show that composite number is prime number and prime number also is composite number. There is no difference between composite number and prime number. Except that 1 is not both prime number and composite number (It can also be said that 1 is both prime number and composite number), we know that natural number is not prime number and composite number. This is like human having man or woman, except two-sex human. If two-sex human metaphor as 1, men as prime number and women as composite number, the two equalities above become like as:

$$\text{Man} + \text{Woman} = \text{Man} + \text{Man}$$

$$\text{Woman} + \text{Woman} = \text{Man} + \text{Man}$$

In this way, woman is man and man is woman. There is no difference between woman and man. Also, pupil can understand this is a mistake in mathematical logic. It is too joking that this absurdity is proved by high-advanced mathematical theory. It is not like that someone said only one step of the achievement in research on (1 + 2) to (1 + 1). This shows that the research route taking (1 + 6) as starting point has come to a dead end. However, the research route taking (9 + 9) as starting point has not come to end, but there is one step, i.e. (2 + 2), to go if someone insists in continuing.

Goldbach conjecture is a general form of problem. If we prove it, we

must grasp the rule of development of (1 + 1) solution of >2 even number. At the time of proving Goldbach conjecture, we can not use old method in the past and must create a new proof method. The author use the analysis—induction method (Also, it can be called as trend analysis method) to make the full analysis of (1 + 1) solution of >2 even number systematically, and induced four specific special rules. At last, he synthesized one general rule as whole. Goldbach conjecture has generally been proved to be completely tenable.

2 Proof of Even Number Goldbach Conjecture

Even number Goldbach conjecture: Each >2 even number is the sum of two prime numbers.

In order to grasp the rule of development of (1 + 1) solution of >2 even number, we made a systematic investigation in (1 + 1) solution of >2 even number from known to unknown, from specific to general and from small to big. In order to facilitate investigation and analysis, we supposed 2N to be >2 even number, (1 + 1) to be (1 + 1) solution of 2N even number, π (1 + 1) to be number of (1 + 1) solution of 2N even number.

1. The results of detail investigation in (1 + 1) solution of 2N=4 ~ 100 are shown in table 1.

2. The results of general investigation in (1 + 1) solution of 2N=100 ~ 1000 are shown in Table 2.

3. The result of general investigation in (1 + 1) solution of 2N=1000 ~ 5000 are shown in Table 3.

Based on Table 1 ~ 3, it can be seen that (1 + 1) solution of >2 even number has four specific special rules as follows:

1. Even prime number 2 only makes up (1 + 1) solution of even number 4: 4=2 + 2. This is unique. Even prime number 2 can not make up (1 + 1) solution of >4 even number. (1 + 1) solution of >4 even number is composed of odd prime number only.

2. Suppose N to equal prime number P, 2N even number is 2P-type even number. Any prime numbers P can make up 2P-type even number. All 2P-type even numbers have (1 + 1) solution of one equal prime number: 2N=2P=P + P. 2P-type even number 4 and 6 have (1 + 1) solution of only one equal prime number. Expect (1 + 1) solution of one equal prime number, >6 2P-type even number has (1 + 1) solution of one or more than one unequal prime numbers.

3. Suppose P_1 and P_2 to be two unequal odd prime numbers, $P_2 > P_1$, and α is positive integer, P_1 and P_2 , in (1 + 1) solution $2N=P_1 + P_2$ of unequal prime number of >6 even number, have the following relation:

$$P_1 = N - \alpha$$

$$P_2 = N + \alpha$$

P_1 and p_2 are symmetric each other, centering on mean N of 2N even number,

and take α as symmetric distance. If N is even number, α is odd number. If N is odd number, α is even number. Therefore, P_1 and P_2 have the rule of "even number centering and odd number symmetry" and "odd number centering and even number symmetry". Two prime number P_1 and P_2, making up (1 + 1) solution of unequal prime number of >6 even number, are prime number pair taking N as symmetric center. Based on existence in symmetric prime number pair, we can use the method for solving binary linear indefinite equation to derive (1 + 1) solution of unequal prime number of >6 2N even number. There is how many prime number pairs taking N as symmetric center, and then 2N even number has how many (1 + 1) solutions of unequal prime number. The steps of solving (1 + 1) solution of unequal prime number of definite even numbers greater than 6 are as follows:

(1) Sort out odd prime numbers from 2N even numbers.

(2) Divide [2, 2N] interval into [2, N] interval and [N, 2N] interval.

(3) Substitute odd prime numbers in [2, N] interval into P_1 in Equation P_2=2N-P_1, and check whether P_2 exist or not. If P_2 exists, it and P_1 make up one symmetric prime number pair, so 2N has one (1 + 1) solution. If P_2 does not exist, P_1 can not make up one symmetric prime number pair and is eliminated. We don't continue in such a way until all symmetric prime number pairs of P_1 - P_2 are derived. On the contrary, substitute prime numbers in [N, 2N] interval into P_2 in Equation P_1=2N-P_2, check whether P_1

exists or not, and find out all symmetric prime number pairs of P_1-P_2, so that all (1 + 1) solution of unequal prime number of 2N even number. In fact, (1 + 1) solution of equal prime number of 2P-type even number is the special form (P_1=P_2, α=0) of (1 + 1) solution of unequal prime number of 2N even number only, and can be derived together.

4. As for even numbers greater than 2, in which there are four even numbers, i.e. 4, 6, 8 and 12, they have only one (1 + 1) solution respectively, but other even numbers have two or more than two (1 + 1) solutions. Number of (1 + 1) solution is 1 ~ 9 for 2N=4 ~ 100, 6 ~ 48 for 2N=100 ~ 1000 and 30 ~ 104 for 2N=1000 ~ 5000, but there is an irregular change within a certain limit partially. The general trend of development is that number of (1 + 1) solution of 2N even number increases quickly with a gradual growing of 2N even number. Therefore, so long small even number greater than 2 (Example: ≤12) has (1 + 1) solution, greater even number (Example: ≥14) has at least two or more than two (1 + 1) solution. Meanwhile, with growing of even number, the general trend of development in aspects of number of (1 + 1) solution of even number is to increase gradually.

Based on four specific special rules of (1 + 1) solution of 2N even number stated above, we can obtain one general rule as whole: Each >2 even number has at least one (1 + 1) solution. Therefore, even number Goldbach

conjecture has been proved:

$$2N=P_1+P_2,\quad P_1\leqslant P_2.$$

3 Proof of Odd Number Goldbach Conjecture

Odd number Goldbach conjecture: Each >5 odd number is the sum of three prime numbers.

Proof: In fact, odd number Goldbach conjecture is the deduction of even number Goldbach conjecture. Therefore, if the latter has been proved, the former is easy to be proved. Suppose odd number M to be greater than 5, q is prime number and M-2>q>2, M-q is even number greater than 2. So, we have:

$$M-q=P_1+P_2,\quad P_1\leqslant P_2.$$

i.e.
$$M=P_1+P_2+q.$$

When M is greater than 7, M has two or more than two (1 + 1 + 1) solution. The greater M is, the more solution is. Odd number Goldbach conjecture has been proved.

4 Conclusions

The author has made a full analysis of (1 + 1) solution of even number greater than 2 systematically, from known to unknown, from specific to general and from small to big, and induced four specific special rules for (1 + 1) solution of 2N even number. At last, he summed up one general rule as

whole: Each >2 even number has at least one (1 + 1) solution. Therefore, we have generally proved "Each >2 even number is the sum of two prime numbers":

$$2N=P_1+P_2,\quad P_1\leqslant P_2.$$

According to the proof of even number Goldbach conjecture, "Each >5 odd number is the sum of three prime numbers" has been proved also: Suppose M to be >5 odd number, q be prime number and M-2>q>2, M-q is >2 even number. So, we have:

$$M-q=P_1+P_2,\quad P_1\leqslant P_2,$$

i.e.

$$M=P_1+P_2+q.$$

The author has generally proved Goldbach conjecture to be completely tenable.

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The Record Table Of （1+1） Solution Of 4～100

Table 1

2N	(1+1)	π (1+1)
4	2+2.	1
6	3+3.	1
8	3+5.	1
10	3+7, 5+5.	2
12	5+7.	1
14	3+11, 7+7.	2
16	3+13, 5+11.	2
18	5+13, 7+11.	2
20	3+17, 7+13.	2
22	3+19, 5+17, 11+11.	3
24	5+19, 7+17, 11+13.	3
26	3+23, 7+19, 13+13.	3
28	5+23, 11+17.	2
30	7+23, 11+19, 13+17.	3
32	3+29, 13+19.	2
34	3+31, 5+29, 11+23, 17+17.	4
36	5+31, 7+29, 13+23, 17+19.	4
38	7+31, 19+19.	2
40	3+37, 11+29, 17+23.	3
42	5+37, 11+31, 13+29, 19+23	4
44	3+41, 7+37, 13+31	3
46	3+43, 5+41, 17+29, 23+23	4
48	5+43, 7+41, 11+37, 17+31, 19+29	5
50	3+47, 7+43, 13+37, 19+31	4
52	5+47, 11+41, 23+29	3
54	7+47, 11+43, 13+41, 17+37, 23+31	5
56	3+53, 13+43, 19+37	3
58	5+53, 11+47, 17+41, 29+29	4
60	7+53, 13+47, 17+43, 19+41, 23+37, 29+31	6
62	3+59, 19+43, 31+31	3
64	3+61, 5+59, 11+53, 17+47, 23+41	5
66	5+61, 7+59, 13+53, 19+47, 23+43, 29+37	6
68	7+61, 31+37	2
70	3+67, 11+59, 17+53, 23+47, 29+41	5

2N	(1+1)	π (1+1)
72	5+67, 11+61, 13+59, 19+53, 29+43, 31+41	6
74	3+71, 7+67, 13+61, 31+43, 37+37	5
76	3+73, 5+71, 17+59, 23+53, 29+47	5
78	5+73, 7+71, 11+67, 17+61, 19+59, 31+47, 37+41	7
80	7+73, 13+67, 19+61, 37+43	4
82	3+79, 11+71, 23+59, 29+53, 41+41	5
84	5+79, 11+73, 13+71, 17+67, 23+61, 31+53, 37+47, 41+43.	8
86	3+83, 7+79, 13+73, 19+67, 43+43.	5
88	5+83, 17+71, 29+59, 41+47.	4
90	7+83, 11+79, 17+73, 19+71, 23+67, 29+61, 31+59, 37+53, 43+47.	9
92	3+89, 13+79, 19+73, 31+61.	4
94	5+89, 11+83, 23+71, 41+53, 47+47.	5
96	7+89, 13+83, 17+79, 23+73, 29+67, 37+59, 43+53.	7
98	19+79, 31+67, 37+61.	3
100	3+97, 11+89, 17+83, 29+71, 41+59, 47+53.	6

The Record Table Of (1+1) Solution Of 100~1000

Table 2

2N	(1+1)	π (1+1)
100	3+97, 11+89, 17+83, 29+71, 41+59, 47+53.	6
200	3+197, 7+193, 19+181, 37+163, 43+157, 61+139, 73+127, 97+103.	8
300	7+293, 17+283, 19+281, 23+277, 29+271, 31+269, 37+263, 43+257, 59+241, 61+239, 67+233, 71+229, 73+227, 89+211, 101+199, 103+197, 107+193, 109+191, 127+173, 137+163, 149+151.	21
400	3+397, 11+389, 17+383, 27+373, 41+359, 47+353, 53+347, 83+317, 89+311, 107+293, 131+269, 137+263, 149+251, 167+233, 173+227.	15
500	13+487,37+463,43+457,61+439,67+433,79+421, 103+397, 127+373, 151+349, 163+337, 193+307, 223+277, 229+271.	13
600	7+593, 13+587, 23+577, 29+571, 31+569, 37+563, 43+557, 53+547, 59+541, 79+521, 97+503, 101+499, 109+491, 113+487, 137+463, 139+461, 151+449, 157+443, 167+433, 179+421, 181+419, 191+409, 199+401, 211+389, 227+373, 233+367, 241+359, 251+349, 263+337, 269+331, 283+317, 293+307.	32
700	17+683, 23+677, 41+659, 47+653, 53+647, 59+641, 83+617, 101+599, 107+593, 113+587, 131+569, 137+563, 179+521, 191+509, 197+503, 233+467, 239+461, 251+449, 257+443, 269+431, 281+419, 311+389, 317+383, 347+353.	24
800	3+797, 13+787, 31+769, 43+757, 61+739, 67+733, 73+727, 109+691, 127+673, 139+661, 157+643, 181+619, 193+607, 199+601, 223+577, 229+571, 277+523, 313+487, 337+463, 367+433, 379+421.	21
900	13+887, 17+883, 19+881, 23+877, 37+863, 41+859, 43+857, 47+853, 61+839, 71+829, 73+827, 79+821, 89+811, 103+797, 113+787, 127+773, 131+769, 139+761, 149+751, 157+743, 167+733, 173+727, 181+719, 191+709, 199+701, 223+677, 227+673, 239+661, 241+659, 257+643, 269+631, 281+619, 283+617, 293+607, 307+593, 313+587, 331+569, 337+563, 353+547, 359+541, 379+521, 397+503,	48

	401+499, 409+491, 421+479, 433+467, 439+461, 443+457.	
1000	3+997, 17+983, 23+977, 29+971, 47+953, 53+947, 59+941, 71+929, 89+911, 113+887, 137+863, 173+827, 179+821, 191+809, 227+773, 239+761, 257+743, 281+719, 317+683, 347+653, 353+647, 359+641, 353+647, 359+641, 383+617, 401+599, 431+569, 443+557, 479+521, 491+509.	30

The Record Table Of (1+1) Solution Of 1000～5000

Table 3

2N	(1+1)	ж (1+1)
1000	3+997, 17+983, 23+977, 29+971, 47+953, 53+947, 59+941, 71+929, 89+911, 113+887, 137+863, 173+827, 179+821, 191+809, 227+773, 239+761, 257+743, 281+719, 317+683, 347+653, 353+647, 359+641, 353+647, 359+641, 383+617, 401+599, 431+569, 443+557, 479+521, 491+509.	30
2000	3+1997, 7+1993, 13+1987, 67+1933, 127+1873, 139+1861, 199+1801, 211+1789, 223+1777, 241+1759, 277 +1723, 307+1693, 331+1669, 337+1663, 373+1627, 379+1621, 421+1579, 433+1567, 457+1543, 541+1459, 547+1453, 571 +1429, 577+1423, 601+1399, 619+1381, 673+1327, 709+1291, 751+1249, 769+1231, 787+1213, 829 +1171, 877+1123, 883+1117, 907+1093, 937+1063, 967+1033, 991+1009.	37

2N	(1+1)	π (1+1)
3000	29+2971, 31+2969, 37+2963, 43+2957, 47+2953, 61+2939, 73+2927, 83+2917, 97+2903, 103+2897, 113+2887, 139+2861, 149+2851, 157+2843, 163+2837, 167 +2833, 181+2819, 197+2803, 199+2801, 211+2789, 223+2777, 233+2767, 251+2749, 269+2731, 271+2729, 281+2719, 293 +2707, 307+2693, 311+2689, 313+2687, 317+2683, 337+2663, 353+2647, 367+2633, 379+2621, 383 +2617, 409+2591, 421+2579, 443+2557, 449+2551, 457+2543, 461+2539, 479+2521, 523+2477, 541+2459, 563+2437, 577 +2423, 601+2399, 607+2393, 617+2383, 619+2381, 643+2357, 653+2347, 659+2341, 661+2339, 691 +2309, 719+2281, 727+2273, 733+2267, 757+2243, 761+2239, 787+2213, 797+2203, 821+2179, 839+2161, 857+2143, 859 +2141, 863+2137, 887+2113, 911+2089, 919+2081, 937+2063, 947+2053, 971+2029, 983+2017, 997+2003, 1013+1987, 1021 +1979, 1049+1951, 1051+1949, 1069+1931, 1087+1913, 1093+1907, 1123+1877, 1129+1871, 1153+1847, 1213+1787, 1217+1783, 1223+1777, 1259+1741, 1277+1723, 1279+1721, 1291+1709, 1301+1699, 1303+1697, 1307+1693, 1373+1627, 1381+1619, 1399+1601, 1429+1571, 1433+1567, 1447+1553, 1451+1549, 1489+1511.	104

2N	(1+1)	π (1+1
4000	11+3989, 53+3947, 71+3929, 83+3917, 89+3911, 137+3863, 149+3851, 167+3833, 179+3821, 197+3803, 233+3767, 239+3761, 281+3719, 383+3617, 419+3581, 443 +3557, 461+3539, 467+3533, 509+3491, 587+3413, 593+3407, 641+3359, 653+3347, 677+3323, 701+3299, 743+3257, 797 +3203, 809+3191, 863+3137, 881+3119, 911+3089, 977+3023, 1031+2969, 1061+2939, 1091+2909, 1097+2903, 1103+2897, 1163+2837, 1181+2819, 1223+2777, 1259+2741, 1289+2711, 1301+2699, 1307+2693, 1367+2633, 1409+2591, 1451+2549, 1523+2477, 1553+2447, 1559+2441, 1583+2417, 1601+2399, 1607+2393, 1619+2381, 1667+2333, 1733+2267, 1787+2213, 1847+2153, 1871+2129, 1889+2111, 1901+2099, 1913+2087, 1931+2069, 1973+2027, 1997+2003.	65
5000	7+4993, 13+4987, 31+4969, 43+4957, 67+4933, 97+4903, 139+4861, 199+4801, 211+4789, 241+4759, 271+ 4729, 277+4723, 337+4663, 349+4651, 379+4621, 397+4603, 409+4591, 433+4567, 439+4561, 487+4513, 577+4423, 643 +4357, 661+4339, 673+4327, 727+4273, 739+4261, 757+4243, 769+4231, 823+4177, 907+4093, 997 +4003, 1033+3967, 1069+3931, 1093+3907, 1123+3877, 1153+3847, 1231+3769, 1291+3709, 1303+3697, 1327+3673, 1429+3571, 1453+3547, 1459+3541, 1471+3529, 1483+3517, 1489+3511, 1531+3469, 1543+3457, 1567+3433, 1609+3391, 1627+3373, 1657+3343, 1669+3331, 1693+3307, 1699+3301, 1741+3259, 1747+3253, 1783+3217, 1831+3169, 1879+3121, 1933+3067, 1951+3049, 1999+3001, 2029+2971, 2083+2917, 2113+2887, 2143+2857, 2203+2797, 2251+2749, 2269+2731, 2281+2719, 2287+2713, 2293+2707, 2311+2689, 2341+2659, 2383+2617.	76

四色猜想的数理逻辑法的直接证明*

内容提要：本文作者创造了一种新的相邻区域段的着色方法，把相邻区域段划分为四大类十二类二十一个类型，然后通过数理逻辑的穷举法论证，用手工就简单明了地直接证明了四色猜想一般成立。

关键词：四色猜想 相邻区域段 二十一个类型 数理逻辑法 直接证明 一般成立

I、序言

地图着色的四色猜想，又称为四色问题，是拓扑学和图论中的一个著名问题。它自 1840 年由德国数学家默比乌斯（A.F.Mobius）通过经验概括提出来以后，一百多年来世界上的许多数学家和其他人对它进行了多方研究。直到 1976 年，两位美国的数学家阿佩尔（Kenneth Appel）和黑肯（Wolfgang Haken）使用三台 IBM360 型超高速电子计算机进行了一千二百多个小时运算，做了一百亿个判断，才得到了一个人机证明。阿佩尔和黑肯在发表他们的成果时，附了一份 460 页的缩微检验表。可是，1981 年施密特对阿佩尔等人的计算程序中的 40%进行了检验，发现了 14 处小错，1 处大错。因此，数学家们怀疑阿佩尔等人用机器证明的可相信程度，四色猜想又回归为一个谜。虽然阿佩尔和黑肯于 1988 年

* 本文已于 2001 年 5 月 2 日在网上向世界公开，网址为：http://limhonggao.home.chinaren.com/zhengming.htm；并于当月 23 日在我国进行了版权登记，作品登记号为作登字：19-2001-A-017 号。

发表文章，说明发现的错误完全可以改正，不是根本性的错误，可是其余 60%的计算程序是否存在更多、更大、甚至是无法改正的错误呢？能不能消除计算程序的错误，阿佩尔等人的人机证明面临着严峻的考验。纵使阿佩尔等人检查出并改正了全部计算程序的错误，那么复杂的人机证明的方法就是最好的证明方法吗？它就是唯一的证明方法吗？答案都是否定的。数学家们由于对计算机不够信任，还是想抛开计算机，仍然希望能用一般演绎的方法去证明四色猜想。

本文表明黑肯的判断“肯定不会对四色猜想给出一个非机器证明”是错误的，四色猜想的证明也并不是像有的人所说的那样，“不能单独用手工来完成，而必须靠人机结合才能给予解决。”[2] 本文作者创造了一种新的相邻区域段的着色方法，把相邻区域段划分为四大类十二类二十一个类型，然后通过数理逻辑的穷举法论证，用手工就简单明了地直接证明了四色猜想一般成立。

II、四色猜想的证明

四色猜想是指画在平面或球面上的每张地图都可以用四种颜色着色，使得有公共边界的每两个相邻区域的颜色都不相同。

所谓“四色”是指可以任意设定的四种颜色，例如红、橙、黄、绿四种颜色。它们没有固定的次序，即颜色的次序也可以任意设定。

四色猜想所说的两个相邻区域，是指彼此之间有一条公共分界线的两个区域。同时，这里所说的区域是连成一片的，而不包括一个区域分成几个部分的情况。[2]

对于不超过四个区域的地图或地图的单独部分的着色，所用的颜色显然不会超过四种，就可以保证每两个相邻区域的颜色不相同，因而无需进行证明。

对于超过四个区域的地图或地图的单独部分的着色，为了证明四色猜想，首先必须创造出一种新的方法，以便将没有分布规律的全部区域构成一个相互关联的体系。在一张地图内的许多区域中，可以任意指定一个区域作为相对的中心区域，与它毗连的区域当然为相对的外围相邻区域（以下仍简称相邻区域）。这种相对的中心区域和相邻区域是动态的：当指定一个区域为相对的中心区域时，与它毗连的区域则为它的相邻区域；当指定另一个前面的相邻区域为相对的中心区域时，前面作为相对的中心区域就和其他相邻区域一样，变成这个相对的中心区域的相邻区域了。这样逐步转换相对的中心区域，就可以把整张地图的全部区域，组构成一个一个相互关联的相邻区域段。所谓相邻区域段，就是由一个相对的中心区域、一个或多个相互毗连的未着色的相邻区域、以及可能存在于未着色相邻区域一端或两端的一个已着色相邻区域的共同组构。相邻区域段的定义表明，一个相对的中心区域和一个或多个相互毗连的未着色的相邻区域是构成相邻区域段的必要因素；而在未着色相

邻区域一端或两端的已着色的一个相邻区域，可以存在，也可以不存在，它（它们）只是相邻区域段划分大类的因素。未着色相邻区域的个数，总的说来可以分为只有一个、2n 个和 2n+1 个（n>0，整数）三类。当有一个以上未着色的相邻区域时，它们必须相互毗连，才能划为同一个相邻区域段；否则不能划为同一个相邻区域段。不相毗连的未着色的相邻区域则构成不同的相邻区域段。因此，对一个相对的中心区域来说，未着色的相邻区域可能相互毗连在一起，构成一个相邻区域段；未着色的相邻区域也可能被分割成多段，构成多个相邻区域段。相反，有多个已着色的相邻区域相互毗连时，只有与未着色相邻区域的一端或两端直接毗连的那一个已着色的相邻区域，才能与未着色相邻区域一起划为同一个相邻区域段；其它已着色的相邻区域与该相邻区域段无关，不能划入该相邻区域段。从相邻区域的排列关系看，在理论上相邻区域段可以呈封闭状排列，也可以呈非封闭状排列。当一个或多个未着色的相邻区域和一端的一个已着色的相邻区域组构成的相邻区域段，围绕相对的中心区域呈封闭状排列时，也可以把已着色的相邻区域看作位于未着色相邻区域的另一端（示意图 7、9、11）。同理，当一个或多个未着色的相邻区域和两端的一个已着不同颜色的相邻区域组构成的相邻区域段，围绕相对的中心区域呈封闭状排列时，两端已着不同颜色的相邻区域便相互毗连在一起了（示意图 16、18、20）。对一个相邻区域段来说，除了上述未着色和已着色的相邻区域以外，其它任何区域都与该相邻区域段

没有任何关系，均不必考虑，在示意图中便可以一律抛开，从而把地图上杂多的区域变为区域有限的一个一个相邻区域段。

归纳上述，相邻区域段的分类原则如下：

1、根据相邻区域相互毗连的关系可以分为：相邻区域均未着色的，一端相邻区域已着色的，两端相邻区域已着相同颜色的和两端相邻区域已着不同颜色的四大类；

2、根据未着色相邻区域的个数可以分为：1 个的，2n 个的和 2n+1 个的三类；

3、根据相邻区域段的排列方式又可以分为：呈封闭状排列的和呈非封闭状排列的两种类型。

在根据相邻区域相互毗连的关系划分的四大类相邻区域段中，其中两端相邻区域已着相同颜色的相邻区域段大类，只能呈非封闭状排列，而显然不能呈封闭状排列。

根据上述相邻区域段的分类原则，便可以将所有地图在理论上可能存在的相邻区域段划分为二十一个类型（表 1）。通过对二十一个类型相邻区域段的着色论证，便可以解决下列三个问题：

1、因为一张地图除了单独部分以外，由它的相邻区域段的总体构成，所以只要每个相邻区域段内的着色不超过四种颜色，则相邻区域段的总体着色就不会超过四种颜色，即整张地图的着色就不会超过四种颜色；

2、因为所有地图的相邻区域段都不会超出上述二十一个类型，所以在二十一个类型的相邻区域段中，只要每一个类型相邻区域段的着色不超过四种颜色，则二十一个类型相邻区域段的总体着色就不会超过四种颜色，所有地图的着色也不会超过四种颜色，即每张地图都可以用四种颜色着色；

3、因为各相邻区域的着色，只与所在相邻区域段内的相邻区域有关系，而与其它区域没有关系，所以只要每一个相邻区域段内有公共边界的每两个相邻区域的颜色不相同，则在二十一个类型相邻区域段的总体上使得有公共边界的每两个相邻区域的颜色都不相同，即每张地图有公共边界的每两个相邻区域的颜色都不相同。

在论证每个类型相邻区域段的着色不超过四种颜色之前，需要设定颜色的次序：在每个相邻区域段中，相对的中心区域所着的颜色为第 1 种颜色，相邻区域所着的颜色可以按照着色的先后分别为第 2 种颜色、第 3 种颜色和第 4 种颜色。因此，这样设定的颜色次序是动态的，各个相互毗连的相邻区域段的颜色次序都不相同。然后，按照顺序逐一证明二十一个类型相邻区域段的着色都不超过四种颜色。

1、一个未着色的相邻区域呈封闭状排列的相邻区域段：未着色的相邻区域可以用第 2 种颜色着色，该相邻区域段共用颜色两种（示意图 1)。若第 1 种颜色未确定，则有四种颜色可以选择（下

同）。若第 1 种颜色已确定，则第 2 种颜色有三种选择。

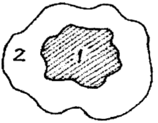

示意图 1

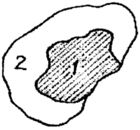

示意图 2

2、 一个未着色的相邻区域呈非封闭状排列的相邻区域段：未着色的相邻区域也可以用第 2 种颜色着色，共用颜色也是两种（示意图 2）。颜色次序及其排列的关系类似 1 类型相邻区域段。

3、2n 个未着色的相邻区域呈封闭状排列的相邻区域段：未着色的相邻区域可以用第 2 种颜色与第 3 种颜色相间着色，共用颜色三种（示意图 3）。第 1 种颜色已确定，第 2 和第 3 种颜色按排列公式 P_3^2 可知有六种选择。

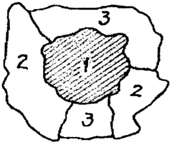

示意图 3

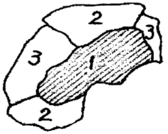

示意图 4

4、2n 个未着色的相邻区域呈非封闭状排列的相邻区域段：未着色的相邻区域也可以用第 2 种颜色与第 3 种颜色相间着色，共用颜色也是三种（示意图 4）。颜色次序及其排列的关系类似 3 类型相邻区域段。

5、2n+1 个未着色相邻区域呈封闭状排列的相邻区域段：前面 2n 个未着色的相邻区域，可以用第 2 种颜色与第 3 种颜色相间着色，最后一个未着色的相邻区域用第 4 种颜色着色，共用颜色四种（示意图 5）。颜色次序及其排列的关系基本上类似 3 类型相邻区域段，着色时第 4 种颜色至少必须用上一次，而着第 4 种颜色的区域可以不同。

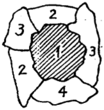

示意图 5

示意图 6

6、2n+1 个未着色相邻区域呈非封闭状排列的相邻区域段：未着色的相邻区域用第 2 种颜色与第 3 种颜色相间着色即可，共用颜色三种（示意图 6）。颜色次序及其排列的关系类似 3 类型相邻区域段。

7、一端相邻区域已着色，未着色相邻区域只有一个，呈封闭状排列的相邻区域段：设一端相邻区域所着的颜色为第 2 种颜色（后面五个类型相邻区域段相同），未着色的相邻区域可以用第 3 种颜色着色，共用颜色三种（示意图 7）。第 1、2 种颜色已确定，第 3 种颜色有两种选择。

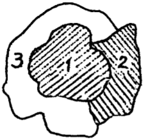

示意图 7

示意图 8

8、一端相邻区域已着色，未着色相邻区域只有一个，呈非封闭状排列的相邻区域段：未着色相邻区域也可以用第 3 种颜色着色，共用颜色也是三种（示意图 8）。颜色次序及其排列的关系类似 7 类型相邻区域段。

9、一端相邻区域已着色，未着色相邻区域有 2n 个，呈封闭状排列的相邻区域段：前面 2n-1 个未着色相邻区域用第 2 种颜色与第 3 种颜色相间着色，最后一个未着色相邻区域用第 4 种颜色着色，共用颜色四种（示意图 9）。第 1、2 种颜色已确定，第 3、4 两种颜色都有两种选择；着色时第 4 种颜色至少必须用上一次，而着第 4 种颜色的区域可以不同。

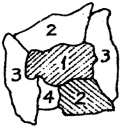

示意图 9

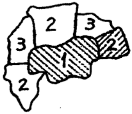

示意图 10

10、 一端相邻区域已着色，未着色相邻区域有 2n 个，呈非封闭状排列的相邻区域段：未着色相邻区域用第 2 种颜色与第 3 种颜色相间着色即可，共用颜色三种（示意图 10）。颜色次序及其排列的关系基本上类似 9 类型相邻区域段，但着色时可以不用第 4 种颜色。

11、 一端相邻区域已着色，未着色相邻区域有 2n+1 个，呈封闭状排列的相邻区域段：未着色相邻区域可以用第 2 种颜色与第 3 种颜色相间着色，共用颜色三种（示意图 11）。颜色次序及其排列的关系类似 10 类型相邻区域段。

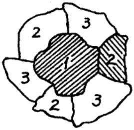

示意图 11

示意图 12

12、 一端相邻区域已着色，未着色相邻区域有 2n+1 个，呈非封闭状排列的相邻区域段：未着色相邻区域也可以用第 2 种颜色与

第 3 种颜色相间着色，共用颜色也是三种（示意图 12）。颜色次序及其排列的关系类似 10 类型相邻区域段。

13、 两端相邻区域已着相同颜色，未着色相邻区域只有一个，呈非封闭状排列的相邻区域段：设两端相邻区域所着的颜色为第 2 种颜色（后面两个类型相邻区域段相同），未着色的相邻区域可以用第 3 种颜色着色，共用颜色三种（示意图 13）。颜色次序及其排列的关系类似 7 类型相邻区域段。

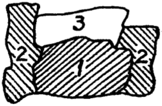

示意图 13

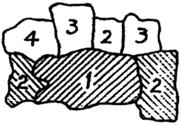

示意图 14

14、 两端相邻区域已着相同颜色，未着色相邻区域有 2n 个，呈非封闭状排列的相邻区域段：前面 2n-1 个未着色相邻区域用第 2 种颜色与第 3 种颜色相间着色，最后一个未着色相邻区域用第 4 种颜色着色，共用颜色四种（示意图 14）。颜色次序及其排列的关系类似 9 类型相邻区域段。

15、 两端相邻区域已着相同颜色，未着色相邻区域有 2n+1 个，呈非封闭状排列的相邻区域段：未着色相邻区域用第 2 种颜色与第 3 种颜色相间着色即可，共用颜色三种（示意图 15）。颜色次序及其排列的关系类似 10 类型相邻区域段。

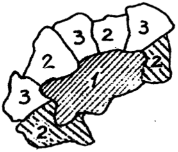

示意图 15

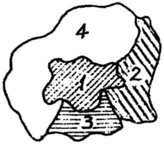

示意图 16

16、 两端相邻区域已着不同颜色，未着色相邻区域只有一个，呈封闭状排列的相邻区域段：设两端相邻区域所着的颜色分别为第 2 种颜色和第 3 种颜色（后面五个类型相邻区域段相同），未着色相邻区域用第 4 种颜色着色，共用颜色四种（示意图 16）。第 1、2、3 种颜色已确定，第 4 种颜色也随着被限定，没有选择的余地。

17、 两端相邻区域已着不同颜色，未着色相邻区域只有一个，呈非封闭状排列的相邻区域段：未着色相邻区域也用第 4 种颜色着色，共用颜色也是四种（示意图 17）。颜色次序及其排列的关系类似 16 类型相邻区域段。

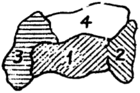

示意图 17

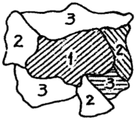

示意图 18

18、 两端相邻区域已着不同颜色，未着色相邻区域有 2n 个，呈封闭状排列的相邻区域段：未着色相邻区域可以用第 2 种颜色与第 3 种颜色相间着色，共用颜色三种（示意图 18）。颜色次序类似 16 类型相邻区域段，但着色时可以不用第 4 种颜色。

19、 两端相邻区域已着不同颜色，未着色相邻区域有 2n 个，呈非封闭状排列的相邻区域段：未着色相邻区域也可以用第 2 种颜色与第 3 种颜色相间着色，共用颜色也是三种（示意图 19）。颜色次序及其排列的关系类似 18 类型相邻区域段。

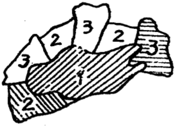

示意图 19

示意图 20

20、 两端相邻区域已着不同颜色，未着色相邻区域有 2n+1 个，呈封闭状排列的相邻区域段：前面 2n 个未着色相邻区域用第 2 种

颜色与第 3 种颜色相间着色，最后一个未着色相邻区域用第 4 种颜色着色，共用颜色四种（示意图 20）。颜色次序及其排列的关系基本上类似 18 类型相邻区域段，但第 4 种颜色至少必须用上一次，而着第 4 种颜色的区域可以不同。

21、 两端相邻区域已着不同颜色，未着色相邻区域有 2n+1 个，呈非封闭状排列的相邻区域段：前面 2n 个未着色的相邻区域也用第 2 种颜色与第 3 种颜色相间着色，最后一个未着色相邻区域用第 4 种颜色着色，共用颜色也是四种（示意图 21）。颜色次序及其排列的关系类似 20 类型相邻区域段。

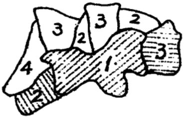

示意图 21

上述已经系统证明，所有地图在理论上可能存在的二十一个类型的相邻区域段中，每一个类型的相邻区域段的着色都不超过四种颜色。显然，二十一个类型的相邻区域段的总体着色也不会超过四种颜色。

此外，在地图着色的实践过程中，地图上可能会出现一些被包围的未着色的区域。那些被包围的未着色的区域都可以当作单独的部分进行着色。当被包围的未着色的区域个数比较少时，在实质上就是上述某些

类型相邻区域段的特殊形式，无需再行论证，从直观就可以看出，对每个被包围部分的着色，共用颜色都不会超过四种。例如：

1、相对的中心区域包围一个或一个以上未着色的相邻区域，在实质上就是第 2、4、6 中的某一个类型相邻区域段的特殊形式：未着色的相邻区域可以分别用第 2 种颜色至第 4 种颜色着色，共用颜色分别为两种至四种；

2、相对的中心区域与一个已着色相邻区域的两部分相互毗连时，如果它们所包围的未着色相邻区域只有一个，在实质上就是第 8 或第 13 类型相邻区域段的特殊形式：未着色相邻区域可以用第 3 种颜色着色，共用颜色三种（如中国与俄罗斯之间的蒙古）；如果被它们包围而且为它们共有的未着色相邻区域有两个或两个以上，在实质上就是第 10 或第 12（也可以看作第 14 或第 15）类型相邻区域段的特殊形式：未着色相邻区域可以用第 3 种颜色与第 4 种颜色相间着色，共用颜色四种（如中国与印度之间的尼泊尔、锡金、不丹）；

3、一个相对的中心区域与两个已着色的相邻区域包围一个未着色相邻区域，在实质上就是第 17 类型相邻区域段的特殊形式：未着色相邻区域用第 4 种颜色着色，共用颜色四种（如赞比亚与坦桑尼亚和莫桑比克包围的马拉维）。

如果还有其它被包围关系的未着色区域，例如第 19 和 21 类型相邻

区域段的特殊形式，或者被包围区域的个数比较多，而且直观难以做出四色判断时，仍然可以把每部分被包围的全部未着色的区域当作该张地图的一个单独部分，在已着色的相邻区域中确定一个新的相对的中心区域，划分新的相邻区域段，按照一定的相邻区域段类型着色，一步一步地扩大，直至完成全部被包围的未着色区域的着色。因为相邻区域段不会超出上述二十一个类型，每个类型相邻区域段所着的颜色都不会超过四种，所以全部被包围区域所着的颜色也不会超过四种。

最后，通过世界地图和中国地图的着色实践，证明上述论证是正确而且可行的。两张地图的着色资料，分别详见表 2、表 3 及世界地图和中国地图着色的说明。

III、结论

作者在本文里首先创造了一种新的相邻区域段的着色方法。所谓相邻区域段，就是由一个相对的中心区域、一个或多个相互毗连的未着色的相邻区域、以及可能存在于未着色相邻区域一端或两端的一个已着色相邻区域的共同组构。这种相邻区域段的划分是动态的，中心区域及其相邻区域都是相对的。这样逐步转换相对的中心区域，就可以把整张地图的全部区域，组构成一个一个相互关联的相邻区域段。根据相邻区域相互毗连的关系、未着色相邻区域的个数和相邻区域段的排列方式等原

则，把地图中在理论上可能存在的相邻区域段总共划分为四大类十二类二十一个类型。所有地图的相邻区域段都不会超出二十一个类型。每一个相邻区域段的着色都是独立自足的，即每一个相邻区域的着色，只与本相邻区域段内的相邻区域有关系，而与本相邻区域段以外的其它区域没有任何关系。然后在各相邻区域段的着色之前，设定动态的颜色次序，用穷举法证明二十一个类型中的每一个类型相邻区域段的着色都不会超过四种颜色，就能使每一个类型相邻区域段内有公共边界的每两个相邻区域的颜色都不相同，则二十一个类型相邻区域段的总体着色也就不会超过四种颜色，在二十一个类型相邻区域段的总体上使得有公共边界的每两个相邻区域的颜色都不相同，即所有地图都可以用四种颜色着色，使得每张地图有公共边界的每两个相邻区域的颜色都不相同。这就运用数理逻辑的方法直接证明了四色猜想一般成立。最后，通过世界地图和中国地图的着色实践，证明上述论证是正确而且可行的。

相邻区域段分类着色表

表 1

相邻区域相互毗连的关系	未着色相邻区域的个数	排列方式		相邻区域段类型	着色种数
		呈封闭状	呈非封闭状		
均未着色	1	√		1	2
			√	2	2
	2n	√		3	3
			√	4	3
	2n+1	√		5	4
			√	6	3
一端相邻区域已着色	1	√		7	3
			√	8	3
	2n	√		9	4
			√	10	3
	2n+1	√		11	3
			√	12	3
两端相邻区域已着相同颜色	1		√	13	3
	2n		√	14	4
	2n+1		√	15	3
两端相邻区域已着不同颜色	1	√		16	4
			√	17	4
	2n	√		18	3
			√	19	3
	2n+1	√		20	4
			√	21	4

世界地图着色记录表

表2

序号	相对的中心区域（颜色）	已着色的相邻区域（颜色）	未着色的相邻区域（颜色）	相邻区域段类型号	着色种数
1	中国（红）		俄罗斯（橙），哈萨克斯坦（黄），吉尔吉斯斯坦（橙），塔吉克斯坦（黄），阿富汗（橙），克什米尔（黄），印度（橙），缅甸（黄），老挝（橙），越南（黄）	4	3
2	中国（红）		朝鲜（橙）	2	2
3	朝鲜（橙）		韩国（黄）	2	2
4	老挝（橙）	缅甸（黄），越南（黄）	泰国（绿），柬浦寨（红）	14	4
5	印度（橙）	缅甸（黄）	孟加拉（红）	8	3
6	泰国（绿）		马来西亚（红）	2	2
7	俄罗斯（橙）		挪威（黄），芬兰（绿）	4	3
8	俄罗斯（加里宁格勒）（橙）		波兰（红），立陶宛（绿）	4	3

序号	相对的中心区域（颜色）	已着色的相邻区域（颜色）	未着色的相邻区域（颜色）	相邻区域段类型号	着色种数
9	立陶宛（绿）	波兰（红）	白俄罗斯（黄），拉脱维亚（红）	10	3
10	拉脱维亚（红）	俄罗斯（橙）	爱沙尼亚（黄）	8	3
11	俄罗斯（橙）	白俄罗斯（黄）	乌克兰（绿）	8	3
12	俄罗斯（橙）		格鲁吉亚（黄），阿塞拜疆（绿）	4	3
13	哈萨克斯坦(黄)	吉尔吉斯斯坦（橙）	乌兹别克斯坦（绿）	8	3
14	克什米尔（黄）	阿富汗(橙)，印度(橙)	巴基斯坦（绿）	13	3
15	乌兹别克斯坦(绿)	哈萨克斯坦(黄),阿富汗(橙)	土库曼斯坦（红）	17	4
16	格鲁吉亚(黄)	阿塞拜疆(两部分，绿)	亚美尼亚（橙），土耳其（红）	10	4

序号	相对的中心区域（颜色）	已着色的相邻区域（颜色）	未着色的相邻区域（颜色）	相邻区域段类型号	着色种数
17	乌克兰（绿）	波兰（红）	斯洛伐克（黄），匈牙利（红），罗马尼亚（黄）摩尔多瓦（红）	10	3
18	芬兰（绿）	挪威（黄）	瑞典（红）	8	3
19	波兰（红）	斯洛伐克（黄）	德国（黄），捷克（橙）	10	3
20	德国（黄）	捷克（橙）	荷兰（绿），比利时（橙），法国（绿），瑞士（橙），奥地利（绿）	12	3
21	德国（黄）		丹麦（绿）	2	2
22	匈牙利（红）	奥地利(绿)，罗马尼亚（黄）	列支敦士登(黄)，克罗地亚(绿)、南斯拉夫（橙）	21	4
23	南斯拉夫(橙)	罗马尼亚（黄）	阿尔巴尼亚(绿),马其顿(黄),保加利亚(绿)	12	3
24	法国（绿）	瑞士（橙）	意大利（红）	8	3
25	法国（绿）		摩纳哥（黄）	2	2
26	法国（绿）		西班牙（橙）	2	2
27	西班牙(橙)		葡萄牙（黄）	2	2
28	马其顿（黄）	阿尔巴尼亚(绿),保加利亚(绿)	希腊（橙）	13	3

序号	相对的中心区域（颜色）	已着色的相邻区域（颜色）	未着色的相邻区域（颜色）	相邻区域段类型号	着色种数
29	土耳其（红）	阿塞拜疆（绿）	叙利亚（黄），伊拉克（绿），伊朗（黄）	12	3
30	伊拉克（绿）	叙利亚（黄）	约旦（红），沙特阿拉伯（黄），科威特（红）	12	3
31	叙利亚(黄)		黎巴嫩（绿）	2	2
32	沙特阿拉伯(黄)		也门(红),阿曼(绿),阿联酋(红),卡塔尔(绿)	4	3
33	约旦（红）		以色列（橙）	2	2
34	以色列（橙）		埃及（红）	2	2
35	埃及（红）		利比亚（橙），苏丹（黄）	4	3
36	利比亚（橙）	苏丹（黄）	突尼斯（黄），阿尔及利亚（绿），尼日尔（黄），乍得（绿）	10	3
37	阿尔及利亚(绿)	尼日尔（黄）	摩洛哥（黄），西撒哈拉（红），毛里塔尼亚（黄），马里（红）	10	3
38	苏丹（黄）	乍得（绿）	中非(红)，刚果民(绿)，乌干达(红)，肯尼亚(绿)，埃塞俄比亚(红)，厄立特里亚(绿)	10	3

序号	相对的中心区域（颜色）	已着色的相邻区域（颜色）	未着色的相邻区域（颜色）	相邻区域段类型号	着色种数
39	埃塞俄比亚(红)	厄立特里亚(绿),肯尼亚(绿)	吉布提(橙)，索马里（黄）	14	4
40	马里（红）	毛里塔尼亚（黄），尼日尔（黄）	塞内加尔（橙），几内亚（黄），科特迪瓦（橙），布基纳法索（绿）	14	4
41	几内亚（黄）	科特迪瓦（橙）	塞拉利昂（橙），利比里亚（绿）	10	3
42	乍得（绿）	尼日尔（黄），中非（红）	尼日利亚（红），喀麦隆（黄）	19	3
43	尼日尔(黄)	布基纳法索（绿），尼日利亚（红）	贝宁（橙）	17	4
44	布基纳法索(绿)	科特迪瓦（橙）,贝宁（橙）	加纳（黄），多哥（红）	14	4
45	喀麦隆（黄）	中非（红）	赤道几内亚（橙），加蓬（红），刚果（橙）	12	3

序号	相对的中心区域（颜色）	已着色的相邻区域（颜色）	未着色的相邻区域（颜色）	相邻区域段类型号	着色种数
46	刚果民(绿)	乌干达（红）	安哥拉（红），赞比亚（黄），坦桑尼亚（红），卢旺达（黄）	10	3
47	赞比亚(黄)	安哥拉(红)，坦桑尼亚（红）	纳米比亚（绿），博茨瓦纳（橙），津巴布韦（红），莫桑比克（橙）	14	4
48	博茨瓦纳(橙)	纳米比亚(绿),津巴布韦(红)	南非（黄）	17	4
49	塞内加尔(橙)		冈比亚（黄）	2	2
50	几内亚（黄）	塞内加尔（橙）	几内亚比绍（绿）	8	3
51	南非（黄）	莫桑比克（橙）	斯威士兰（红）	8	3
52	加拿大（红		美国（橙）	2	2
53	美国（橙）		墨西哥（黄）	2	2
54	墨西哥（黄）		危地马拉（绿），伯利兹（红）	4	3
55	危地马拉(绿)		萨尔瓦多（红），洪都拉斯（橙）	4	3

序号	相对的中心区域（颜色）	已着色的相邻区域（颜色）	未着色的相邻区域（颜色）	相邻区域段类型号	着色种数
56	洪都拉斯(橙)		尼加拉瓜（黄）	2	2
57	尼加拉瓜(黄)		哥斯达黎加（红）	2	2
58	哥斯达黎加(红)		巴拿马（橙）	2	2
59	巴拿马（橙）		哥伦比亚（黄）	2	2
60	哥伦比亚（黄）		厄加多尔（红），秘鲁（绿），巴西（红），委内瑞拉（绿）	4	3
61	巴西（红）	委内瑞拉（绿）	圭亚那(黄)，苏里南(绿)，法属圭亚那(黄)	12	3
62	巴西（红）	秘鲁（绿）	玻利维亚（黄），阿根廷（绿），乌拉圭（黄）	12	3
63	玻利维亚（黄）	阿根廷(绿)，巴西（红）	巴拉圭（橙）	17	4
64	玻利维亚（黄）	秘鲁（绿），阿根廷（绿）	智利（红）	13	3

中国地图着色记录表

表3

序号	相对的中心区域（颜色）	已着色的相邻区域（颜色）	未着色的相邻区域（颜色）	相邻区域段类型号	着色种数
1	北京（红）		天津（橙），河北（黄）	3	3
2	河北（黄）		辽宁(绿)，内蒙(红)，山西(绿)，河南(红)，山东(绿)	6	3
3	内蒙（红）	辽宁（绿）	吉林（橙），黑龙江（绿）	10	3
4	河南（红）	山西（绿），山东（绿）	陕西（橙），湖北（绿），安徽（橙）	15	3
5	安徽（橙）	湖北（绿），山东（绿）	江西（黄），浙江（绿），江苏（黄）	15	3
6	江西（黄）	湖北（绿），浙江（绿）	湖南（红），广东（绿），福建（红）	15	3
7	江苏（黄）	浙江（绿）	上海（红）	8	3
8	广东（绿）		香港（红）	2	2
9	湖南（红）	湖北（绿），广东（绿）	重庆（黄），贵州（绿），广西（黄）	15	3
10	重庆（黄）	陕西（橙），贵州（绿）	四川（红）	17	4

11	陕西（橙）	内蒙（红），四川（红）	宁夏（绿），甘肃（黄）	14	4
12	四川（红）	甘肃（黄），贵州（绿）	青海（绿），西藏（黄），云南（橙）	21	4
13	青海（绿）	甘肃（黄），西藏（黄）	新疆（红）	13	3

世界地图和中国地图着色的说明：

1、世界地图和中国地图着色的底图都是按照星球地图出版社 1998 年 4 月出版的正规地图描绘的，比例尺分别为 1:3300 万和 1:600 万。

2、两张地图的着色过程基本一样：在开始的时候，任意确定一个未着色的区域作为相对的中心区域，着上任意一种颜色。这个相对的中心区域所着的颜色，为与这个相对的中心区域有关的相邻区域段的第 1 种颜色。然后，考察该相对的中心区域的相邻区域段的个数，再按照各相邻区域段的类型进行着色。完成与第一个相对的中心区域有关的各相邻区域段的着色以后，再在前面已着色的相邻区域中确定一个作为下一个相对的中心区域。它所着的颜色又作为与这个相对的中心区域有关的相邻区域段的第 1 种颜色。像前面一样，找出与该相对的中心区域有关的相邻区域段的个数，再按照各相邻区域段类型进行着色。如此，逐步转换相对的中心区域，使后面的相邻区域段联系前面的相邻区域段，一步一步地展开，逐渐扩大，直

至将整张地图的全部区域着色完毕。对于地图的各个部分，例如世界地图的着色，如果先对欧洲、亚洲、非洲大陆国家着色，然后再对南、北美洲大陆国家着色，或者相反，在后一部分都需要重新开始确定相对的中心区域，再进行相邻区域段的划分，然后着色。世界地图的着色与中国地图的着色比较，除了相同类型的相邻区域段较多，单独的部分大大增多以外，仅仅多了一些简单的被包围区域。

3、单独的部分，分别任意设定一种或两种颜色，例如世界地图中的日本（绿），菲律宾（橙），新加坡（绿），印尼和巴布亚新几内亚（黄，绿），斯里兰卡（绿），巴林（橙），马达加斯加（黄），澳大利亚（红），新西兰（橙），格陵兰（黄），冰岛（红），英国和爱尔兰（黄，绿），古巴（橙），牙买加（黄），海地和多米尼加（红，黄），波多黎各（橙），特立尼达和多巴哥（红）等（大洋中的其它岛国不再赘述）；中国地图中的台湾（绿），海南（橙），澳门（黄）。

4、被包围国家的着色，根据上述原则有：圣马力诺（黄），梵蒂冈（绿），卢森堡（红），波斯尼亚——黑塞哥维那（黄），蒙古（绿），尼泊尔（黄），锡金（绿），不丹（黄），布隆迪（橙），马拉维（绿），莱索托（绿）等。

5、除了单独部分和被包围的区域以外，世界地图 64 个相邻区域段的着色，其中 63 个相邻区域段都按照各相邻区域段类型的着色种数着色，而第 16 个相邻区域段，本来可以按照该相邻区域段类型的着色种数

（三种颜色）着色，但由于阿塞拜疆分为东、西两部分（这已属例外情况），土耳其如果着绿色，容易与绿色的阿塞拜疆西面部分产生混同的错觉（虽然它们之间没有公共边界，却很靠近），特意将土耳其改着红色，多用一种颜色，还是在四色之内（其实，通过调整颜色排列，用三种颜色也能够使它们明显区别开来）。美国和俄罗斯在大陆上都分成两部分，但对所在相邻区域段的着色都没有影响（实际上，一个区域分成若干个部分是没有关系的，只要首次涉及一个部分着色时，其它各个部分同时都用同一种颜色着色即可）。中国地图 13 个相邻区域段，全部按照相邻区域段类型的着色种数着色。

6、除了单独部分和被包围区域以外，世界地图所有的 64 个相邻区域段归入 12 个相邻区域段类型；中国地图所有的 13 个相邻区域段归入 10 个相邻区域段类型；在两张地图中，呈非封闭状排列的 12 个相邻区域段类型全部都出现了，而呈封闭状排列的第 1、5、7、9、11、16、18、20 八个相邻区域段类型均未出现，第 3 类型相邻区域段也只在中国地图中出现一次，这说明地图上实际的相邻区域段绝大多数呈非封闭状排列，而呈封闭状排列的极少。

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THE DIRECT PROOF OF FOUR-COLOUR CONJECTURE BY METHOD OF MATHEMATICAL LOGIC*

ABSTRACT: The author has pioneered an innovative colouring method for adjacent region sections. The adjacent region section can be divided into four major classes twelve classes twenty-one types. Through demonstration by use the method of exhaustion of mathematical logic, it is direct proved simply, based on manual computation, that the four-colour conjecture is tenable generally.

KEY WORDS: Four-colour conjecture, Adjacent region section, Twenty-one types, Method of mathematical logic, Direct proof, Tenable generally.

1 INTRODUCTION

The four-colour conjecture on colouring of maps, referred to as four-colour problem too, is one well-known problem in topology and graph theory. Since it was put forward by A.F.Mobius, a mathematician in Germany, according to his summing-up experience in 1840, many mathematicians and other mathematics-lovers in the world have made an extensive approach to the four-colour conjecture for more than one hundred years. It was not until 1976 that a human-computer proof of this conjecture was provided by Kenneth Appel and Wolfgang Haken, mathematicians in U.S.A., who used three sets of Model IBM360 high-speed computers to make more than 1200 hours of calculation and 10 billion times of judgement. The achievement in research on the four-colour conjecture, released by Appel and Haken, was enclosed with a mini-copy of check table, 460pp. However, it was found that there were fourteen minor errors and one serious error in 40% of computational process, which Appel et al made a calculation, checked by Schmidt in 1981. In this case, mathematicians felt suspicious of confidence in the result which Appel et al used computers to prove the four-colour conjecture. Since then, the four-colour conjecture becomes a mystery again. In 1988, Appel and Haken published an article to explain these errors found can be quite

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corrected, because they are not an essential mistake. However, mathematicians still hope that a general method of deduction could be used to prove the four-colour conjecture, because they don't have adequate confidence in computer.

This paper indicates that Haken's judgement which the non-computerized proof of four-colour conjecture can not be surely provided was not correct; Unlike in someone's talking, the proof of four-colour conjecture must be done by combination of man with computer rather than only by manual computation[2]. The author has pioneered an innovative colouring method for adjacent region section. The adjacent region section can be divided into four major classes twelve classes twenty-one types. He demonstration by use the method of exhaustion of mathematical logic, direct prove that simply, based on manual computation, the four-colour conjecture is tenable generally.

2 PROOF OF FOUR-COLOUR CONJECTURE

The four-colour conjecture means every map drawn on plane or sphere can be coloured with four colours to make colours in every two adjacent regions with common boundary different.

The so-called four-colour implies four kinds of colours which can be optionally given. For example: red, orange,yellow and green. There is not any fixed-sequence in colouring with them,that is to say,the sequence of colours can be optionally given too.

The two adjacent regions described in the four-colour conjecture are said to be the two regions having common boundary line each other. Meanwhile, the said regions are connected into one, but it is not included that one region is divided into several parts[2].

As for colouring of map with not more than four regions or colouring of separate part on map, the colours used can be not more than four kinds obviously. In this way, it can be ensured that colours in every two adjacent regions are different, thus its proof is not necessary.

In order to prove the four-colour conjecture, pioneering a new method at first is necessary for colouring of map with more than four regions or colouring of separate part on map so as to make up all regions without law of distribution into an interdependent system. As for many regions on one map, one region can be optionally designated as a relative center region, whose adjacent regions, of course, are the relative periphery adjacent-region (which is hereinafter called adjacent region for short). This kind of relative center regions and adjacent regions are dynamic. When one region is designated

as the relative center region, the region adjacent to it is its adjacent region; When the other former adjacent region is designated as the relative center region, the former as the relative center region, same as other adjacent regions, becomes the adjacent regions of this relative center region. Through shifting the relative center region step by step, all regions on the whole map can be made up into one adjacent region section interdependent with the other. The so-called adjacent region section is the common composition of one relative center region, one or more uncoloured adjacent region which is mutually adjacent, and one coloured adjacent region existing possibly at one end or two ends of uncoloured adjacent region. The definition of adjacent region section indicates that one relative center region and one or more uncoloured adjacent region which is mutually adjacent are the necessary factor composing the adjacent region section, but one adjacent region coloured at one end or two ends of uncoloured adjacent region can exist and can not exist too, which (they) is only the factor that the adjacent region section is divided into a main class. Number of uncoloured adjacent regions, generally, can be divided into only three classes: 1, 2n and 2n+1(n>0,integer). At the time of more than one uncoloured adjacent regions, these regions can be divided into the same one adjacent region section on condition that they must be adjacent one another, otherwise they can not be divided into the same one adjacent region section. The uncoloured adjacent regions which are not adjacent one another are made up into different adjacent region sections. In the case of one relative center region, therefore, the uncoloured adjacent regions are possibly adjacent one another to make up into one adjacent region section, also are possibly divided into more sections to compose more adjacent region sections. On the contrary, when more coloured adjacent regions are adjacent each other, only one coloured adjacent region directly adjacent to one end or two ends of uncoloured adjacent region can be divided, together with uncoloured adjacent region, into the same one adjacent region section; The other coloured adjacent regions are independent from this adjacent region section, so can not be fallen into this section. As viewed from the relation of adjacent region arrangement, the adjacent region sections, theoretically, can be closed or unclosed too. When the adjacent region section, which composes of one or more uncoloured adjacent region and one coloured adjacent region at one end, rounding the relative center region takes the shape of closed arrangement, the coloured adjacent region can be considered as the other end locating at uncoloured adjacent region (shown in Fig 7, 9 and

11) too. Similarly, when the adjacent region section, which composes of one or more uncoloured adjacent region and one differently coloured adjacent region at two ends, rounding the relative center region takes the shape of closed arrangement, the differently coloured adjacent regions at two ends are adjoined together, shown in Fig.16,18 and 20. In the case of one adjacent region section, the other regions are independent of this adjacent region section, have no need to be considered and can be got away in the sketch maps, with the exception of uncoloured and coloured adjacent regions above mentioned, so various regions on maps are changed into one and one adjacent region section whose regions are definite.

To sum up, the principles of adjacent region section classification are as follows:

(1)According to mutual adjoining relation of adjacent regions, the adjacent regions can be divided into four major classes, i.e. all uncoloured adjacent regions, coloured adjacent region at one end, same coloured adjacent regions at two ends and differently coloured adjacent regions at two ends;

(2)According to uncoloured adjacent region number, the adjacent regions can be divided into three classes: 1, 2n and 2n+1;

(3)According to forms of adjacent region section arrangement, the adjacent region sections can be divided into two types, i.e. closed arrangement and unclosed arrangement.

In the sections of four major classes of adjacent regions divided by the mutual adjoining relation of adjacent regions, the main class of adjacent region sections in which the adjacent regions at two ends are same coloured take the shape of only unclosed arrangement rather than closed arrangement obviously.

According to the same principles of adjacent region section classification, the adjacent region sections, theoretically, existing possibly on all maps can be divided into 21 types, shown in Table 1. By demonstrating the colouring of 21 types of adjacent region sections, the following three problems can be solved.

(1)Because one map, with the exception of separate parts, is totally composed of its adjacent region sections, the total colouring of adjacent region sections can not be more than four colours so long as the colouring in every adjacent region section is not more than four colours, that is to say, the colouring of the whole map can not be more than four colours;

(2)Because the adjacent region sections on all maps can not be more than 21 types

above mentioned, the total colouring of 21types of adjacent region sections can not be more than four colours and the colouring of all maps can not be more than four colours so long as the colouring of every type in 21 types of adjacent region sections is not more than four colours, that is to say, every map can be coloured with four colours;

(3)Because the colouring of every adjacent region is dependent only of the adjacent regions in the adjacent region section but independent of other regions, the colours in every two adjacent regions with common boundary in 21 types of adjacent region sections, generally, are made to be different so long as the colours in every two adjacent regions with common boundary in every one adjacent region section are different, that is to say, the colours in every two adjacent regions with common boundary on every one map are different.

Prior to demonstrating that the colouring of every type of adjacent region section is not more than four colours, it is necessary that the sequence of colours is given. In every adjacent region section, the colouring of relative center region is the 1st colour; According to the sequence of colouring, the colouring of adjacent regions can be the 2nd, 3rd and 4th colour respectively. The sequence, therefore, which the colour is given in this manner is dynamic, i.e. the sequence of colours in the adjacent region sections adjoining one another are different. And then, it is proved, one by one based on the sequence, that the colouring of 21 types of adjacent region sections is not more than four colours.

Type 1: The adjacent region section in which 1 uncoloured adjacent region takes the shape of closed arrangement: The uncoloured adjacent region can be coloured with the 2nd colour, and two kinds of colours are used in this adjacent region section, shown in Fig.1.If 1st colour is no determine, it has four selections of colour （the following is alike） .If 1st colour is determined, 2nd colour has three selections of colour.

Type 2: The adjacent region section in which 1 uncoloured adjacent region takes the shape of unclosed arrangement: The uncoloured adjacent region can be coloured with the 2nd colour too, in which there are two kinds of colours used in total (Fig.2). Colour order and their arrangement relations are similar to type 1 adjacent region section.

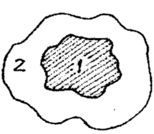


Fig.1 Sketch Map

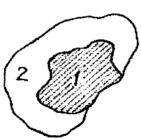


Fig. 2 Sketch Map

Type 3: The adjacent region section in which 2n uncoloued adjacent regions take the shape of closed arrangement: The uncoloured adjacent regions can be coloured alternately with the 2nd and 3rd colour, in which there are three kinds of colours used in total (Fig.3).1st colour is determined, according to the formula for arrangement ($P^2{}_3$) ,it can be known that six kinds of choice are made from the 2nd and 3rd colour.

Type 4: The adjacent region section in which 2n uncoloured adjacent regions take the shape of unclosed arrangement: The uncoloured adjacent regions can be coloured alternately with the 2nd and 3rd colours too, in which there are three kinds of colours used in total (Fig.4).Colour order and their arrangement relations are similar to type 3 adjacent region section.

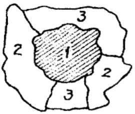


Fig.3 Sketch Map

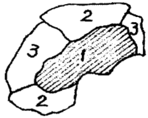


Fig.4 Sketch Map

Type 5: The adjacent region section in which 2n+1 uncoloured adjacent regions take the shape of closed arrangement: The former 2n uncoloured adjacent regions can be coloured alternately with the 2nd and 3rd colour and the last one uncoloured adjacent region is coloured with the 4th colour, in which there are four kinds of colours used in total (Fig.5).Colour order and their arrangement relations are similar to type 3 adjacent region section basically in the time of colouring, 4th colour must be used for once at least, the region coloured with 4th colour can be different.

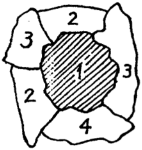


Fig.5 Sketch Map

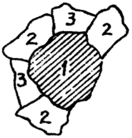


Fig.6 Sketch Map

Type 6: The adjacent region section in which 2n+1 uncoloured adjacent regions take the shape of unclosed arrangement: The uncoloured adjacent regions can be coloured

alternately with the 2nd and 3rd colour, in which there are three kinds of colours used in total (Fig.6).Colour order and their arrangement relations are similar type 3 adjacent region section.

Type 7: The adjacent region section in which the adjacent region at one end has been coloured and there is only 1 uncoloured adjacent region, as closed arrangement: Given that the colour used for the adjacent region at one end is the 2nd colour (the same for the later five types of adjacent region sections), the uncoloured adjacent region can be coloured with the 3rd colour, in which there are three kinds of colours used in total (Fig.7).1^{st} and 2^{nd} colours are determined, 3^{rd} colour has two selection of colour.

Type 8: The adjacent region section in which the adjacent region at one end has been coloured and there is only 1 uncoloured adjacent region, as unclosed arrangement: The uncoloured adjacent region can be coloured with the 3rd colour too, in which there are three kinds of colours used in total (Fig.8).Colour order and their arrangement relations are similar to type 7 adjacent region section.

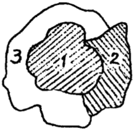


Fig.7 Sketch Map

Fig.8 Sketch Map

Type 9: The adjacent region section in which the adjacent region at one end has been coloured and there are 2n uncoloured adjacent regions, as closed arrangement: The former 2n-1 uncoloured adjacent regions are coloured alternately with the 2nd and 3rd colour and the last one uncoloured adjacent region is coloured with the 4th colour, in which there are four kinds of colours used in total (Fig.9).1^{st} and 2^{nd} colour are determined, 3^{rd} and 4^{th} colour has all two selection of colour in the time of colouring, 4^{th} colour must be used for once at least and region coloured with 4^{th} colour can be different.

Type 10: The adjacent region section in which the adjacent region at one end has been coloured and there are 2n uncoloured adjacent regions, as unclosed arrangement:

The uncoloured adjacent regions can be coloured alternately with the 2nd and 3rd colour, in which there are three kinds of colours used in total (Fig.10).Colour order and their arrangement relations are similar to type 9 adjacent region section basically, 4th colour can be take out in colouring.

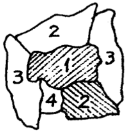


Fig.9 Sketch Map

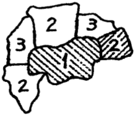


Fig.10 Sketch Map

Type 11: The adjacent region section in which the adjacent region at one end has been coloured and there are 2n+1 uncoloured adjacent regions, as closed arrangement: The uncoloured adjacent regions can be coloured alternately with the 2nd and 3rd colour, in which there are three kinds of colours used in total (Fig.11).Colour order and their arrangement relations are similar to type 10 adjacent region section.

Type 12: The adjacent region section in which the adjacent region at one end has been coloured and there are 2n+1 uncoloured adjacent regions, as unclosed arrangement: The uncoloured adjacent regions can be coloured alternately with the 2nd and 3rd colour too, in which there are three kinds of colours used in total (Fig.12).Colour order and their arrangement relations are similar to type 10 adjacent region section.

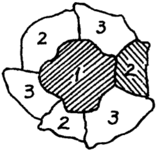


Fig.11 Sketch Map

Fig.12 Sketch Map

Type 13: The adjacent region section in which the adjacent regions at two ends have been coloured with the same colour and there is only 1 uncoloured adjacent region, as unclosed arrangement: Given that the colouring of adjacent regions at two ends is the 2nd colour (the same for the later two types of adjacent region sections), the uncoloured adjacent region can be coloured with the 3rd colour, in which there are three kinds of colours used in total (Fig.13).Colour order and their arrangement relations are similar to type 7 adjacent region section.

Type 14: The adjacent region section in which the adjacent regions at two ends have been coloured with the same colour and there are 2n uncoloured adjacent regions, as unclosed arrangement: The former 2n-1 uncoloured adjacent regions are coloured alternately with the 2nd and 3rd colour and the last one uncoloured adjacent region is coloured with the 4th colour, in which there are four kinds of colours used in total (Fig.14).Colour order and their arrangement relations are similar to type 9 adjacent region section.

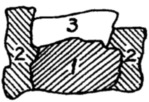


Fig.13 Sketch Map

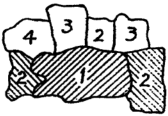
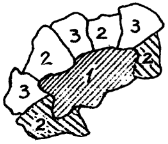


Fig.14 Sketch Map

Type 15: The adjacent region section in which the adjacent regions at two ends have been coloured with the same colour and there are 2n+1 uncoloured adjacent regions, as unclosed arrangement: The uncoloured adjacent regions can be coloured alternately with the 2nd and 3rd colour, in which there are three kinds of colours used in total (Fig.15).Colour order and their arrangement relations are similar to type 10 adjacent region section.

Type 16: The adjacent region section in which the adjacent regions at two ends have been coloured with different colours and there is only 1 uncoloured adjacent region, as closed arrangement: Given that the colouring of adjacent regions at two ends is the 2nd and 3rd colour respectively (the same for the later five types of adjacent region sections), the uncoloured adjacent region is coloured with the 4th colour, in which there are four

kinds of colours used in total (Fig.16).1st,2nd and 3rd colours are determined,4th colour is limited and has not choice at all.



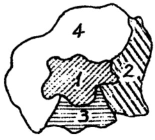
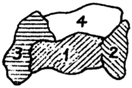


Fig.15 Sketch Map **Fig.16 Sketch Map**

Type 17: The adjacent region section in which the adjacent regions at two ends have been coloured with different colours and there is only 1 uncoloured adjacent region, as unclosed arrangement: The uncoloured adjacent region is coloured with the 4th colour too, in which there are four kinds of colours used in total (Fig.17).Colour order and their arrangement relations are similar to type 16 adjacent region section.

Type 18: The adjacent region section in which the adjacent regions at two ends have been coloured with different colours and there are 2n uncoloured adjacent regions, as closed arrangement: The uncoloured adjacent regions can be coloured alternately with the 2nd and 3rd colour, in which there are three kinds of colours used in total (Fig.18). Colour order is similar to type 16 adjacent region section,4th colour can be take out in colouring.



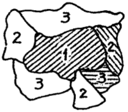


Fig.17 Sketch Map **Fig.18 Sketch Map**

Type 19: The adjacent region section in which the adjacent regions at two ends have been coloured with different colours and there are 2n uncoloured adjacent regions, as unclosed arrangement: The uncoloured adjacent regions can be coloured alternately with

the 2nd and 3rd colour too, in which there are three kinds of colours used in total (Fig.19).Colour order and their arrangement relations are similar to type 18 adjacent region section.

Type 20: The adjacent region section in which the adjacent regions at two ends have been coloured with different colours and there are 2n+1 uncoloured adjacent regions, as closed arrangement: The former 2n uncoloured adjacent regions are coloured alternately with the 2nd and 3rd colour and the last one uncoloured adjacent region is coloured with the 4th colour, in which there are four kinds of colours used in total (Fig.20).Colour order and their arrangement relations are similar to type 18 adjacent region section basically in the time of colouring,4th colour must be used for once at least and the region coloured with 4th colour can be different.

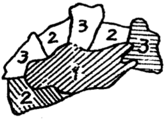


Fig.19 Sketch Map

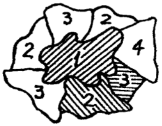


Fig.20 Sketch Map

Type 21: The adjacent region section in which the adjacent regions at two ends have been coloured with different colours and there are 2n+1 uncoloured adjacent regions, as unclosed arrangement: The former 2n uncoloured adjacent regions are coloured alternately with the 2nd and 3rd colour too, and the last one uncoloured adjacent region is coloured with the 4th colour, in which there are four kinds of colours used in total (Fig.21).Colour order and their arrangement relations are similar to type 20 adjacent region section.

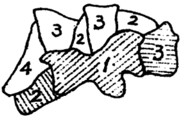


Fig.21 Sketch Map

As stated above, it has been systematically proved that the colouring of every type in 21 types of adjacent region sections, theoretically, existing possibly on all maps is not more than four colours. Obviously, the total colouring of 21 types of adjacent region sections can be not more than four colours too.

In addition, some encircled and uncoloured regions on maps will possibly appear in practice of map colouring. These regions, as a separate part, can be coloured. When number of these regions is less, they are, essentially, a special form of some types of above adjacent regions, so have no need of demonstration again. It can be directly seen that the colouring of every encircled part can be not more than four colours used in total, for examples:

(1) One or more one uncoloured region encircled by the relative center region, in fact, is the special form of one type of adjacent region section in Type 2, 4 and 6 above: The uncoloured adjacent regions can be coloured with the 2nd through 4th colour respectively, whose colours used in total are up to 2 through 4 kinds respectively.

(2) When the two parts, i.e. relative center region and one coloured adjacent region, adjoin one another, they are, essentially, the special form of Type 8 or 13 adjacent region section if there is only one uncoloured adjacent region encircled by them: The uncoloured adjacent region can be coloured with the 3rd colour, whose colours used in total are up to 3 kinds (For example:Mongolia between China and Russia); They are , essentially, the special form of Type 10 or 12 (which can be considered as Type 14 or 15) adjacent region section if there are two or more two uncoloured adjacent regions encircled and shared by them: The uncoloured adjacent regions can be coloured alternately with the 3rd and 4th colour, whose colours used in total are up to 4 kinds (For example: Nepal, Sikkin, Bhutan between China and India).

(3)One uncoloured adjacent region encircled by one relative center region and two coloured adjacent regions, essentially, are the special form of Type 17 adjacent region section: The uncoloured adjacent region is coloured with the 4th colour, whose colours used in total are up to 4 kinds (For example: Malawi encircled by Zambia, Tanzania and Mozambique).

If there are other uncoloured regions with the encircled relation (For example: the special form of Type 19 and 21 adjacent region sections), or number of regions which are encircled is more, all uncoloured regions

encircled in every part, when the four-colour judgement is difficult to be made by direct perception, can be still considered as one separate part on this map; One new relative center region in the coloured adjacent regions is determined and the new adjacent region section is divided; According to the colouring of a certain type of adjacent region section, the uncoloured regions completely encircled are coloured step by step till their colouring is finished. Because the adjacent region sections can be not more than 21 types and the colouring of every type of adjacent region section can be not more than four kinds of colours too, the colours used for the completely-encircled regions can be not more than four kinds.

At last, the practice in the colouring of world map and China map has proved the above demonstration is not only correct and feasible. The information on the colouring of two maps is shown in Table 2~3 and by their descriptions.

3 CONCLUSIONS

The author has pioneered a dynamic dividing method for the adjacent region section. Based on this method, the regions, without law of distribution, on map are made up, step by step, into one and one interdependent adjacent region section; Afterwards, the adjacent region section is determined and the principles of the classification of adjacent region sections are further put forward; And the adjacent region sections, theoretically, existing possibly on map are divided into 21 types in total, then the dynamic sequence of colours is given between adjacent region sections. Meanwhile, the method of exhaustion is used to prove the colouring of each type of adjacent region section can be not more than four colours, so the total colouring of 21 types of adjacent region sections can be not more than four colours too. Because the adjacent region sections on all maps can be not more than 21 types, one map or all maps can be coloured with four colours. In addition, because the colouring of every adjacent region is related only to the adjacent region in its adjacent region section but independent of other regions, the colours of every two adjacent regions with common boundary in each type of adjacent region section are different, that is to say, the total

colouring of every two adjacent regions with common boundary in 21 types of adjacent region sections is made different, i.e. the colours of every two adjacent regions with common boundary on every map are different. The method of mathematical logic has been used to direct prove the four-colour conjecture is tenable generally. At last, the practice in colouring of world map and China map has proved the above proof is not only correct and feasible.

Table 1 Classification Colouring of Adjacent Region Sections

<table>
<tr><th rowspan="2">Mutual Adjoining Relation of Adjacent Regions</th><th rowspan="2">Number of Uncoloured Adjacent Regions</th><th colspan="2">Arrangement Form</th><th rowspan="2">Type of Adjacent Region Section</th><th rowspan="2">Kinds of Colouring</th></tr>
<tr><th>Closed</th><th>Unclosed</th></tr>
<tr><td rowspan="6">Uncoloured</td><td rowspan="2">1</td><td>√</td><td></td><td>1</td><td>2</td></tr>
<tr><td></td><td>√</td><td>2</td><td>2</td></tr>
<tr><td rowspan="2">2n</td><td>√</td><td></td><td>3</td><td>3</td></tr>
<tr><td></td><td>√</td><td>4</td><td>3</td></tr>
<tr><td rowspan="2">2n+1</td><td>√</td><td></td><td>5</td><td>4</td></tr>
<tr><td></td><td>√</td><td>6</td><td>3</td></tr>
<tr><td rowspan="6">Coloured (Adjacent Region at One End)</td><td rowspan="2">1</td><td>√</td><td></td><td>7</td><td>3</td></tr>
<tr><td></td><td>√</td><td>8</td><td>3</td></tr>
<tr><td rowspan="2">2n</td><td>√</td><td></td><td>9</td><td>4</td></tr>
<tr><td></td><td>√</td><td>10</td><td>3</td></tr>
<tr><td rowspan="2">2n+1</td><td>√</td><td></td><td>11</td><td>3</td></tr>
<tr><td></td><td>√</td><td>12</td><td>3</td></tr>
<tr><td rowspan="3">Same Coloured (Adjacent Regions at Two Ends)</td><td>1</td><td></td><td>√</td><td>13</td><td>3</td></tr>
<tr><td>2n</td><td></td><td>√</td><td>14</td><td>4</td></tr>
<tr><td>2n+1</td><td></td><td>√</td><td>15</td><td>3</td></tr>
<tr><td rowspan="6">Differently Coloured (Adjacent Regions at Two Ends)</td><td rowspan="2">1</td><td>√</td><td></td><td>16</td><td>4</td></tr>
<tr><td></td><td>√</td><td>17</td><td>4</td></tr>
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<tr><td></td><td>√</td><td>19</td><td>3</td></tr>
<tr><td rowspan="2">2n+1</td><td>√</td><td></td><td>20</td><td>4</td></tr>
<tr><td></td><td>√</td><td>21</td><td>4</td></tr>
</table>

Table 2 Colouring Records of World Map

No	Relative Center Region (Colour)	Coloured Adjacent Regions (Colour)	Uncoloured Adjacent Regions (Colour)	Type of Adjacent Region Section	Kinds Of Colouring
1	China (red)		Russia(orange) Kazakhstan(yellow) Kyrgzstan(orange) Tajikistan(yellow) Afghanistan(orange) Kashmir (yellow) India (orange) Burma (yellow) Laos (orange) Vietnam(yellow)	4	3
2	China(red)		Korea, North(orange)	2	2
3	Korea,North (orange)		Korea, South(yellow)	2	2
4	Laos (orange)	Burma(yellow) Vietnam(yellow)	Thailand(green) Cambodia(red)	14	4
5	India(orange)	Burma(yellow)	Bangladesh(red)	8	3
6	Thailand (green)		Malaysia(red)	2	2
7	Russia (orange)		Norway(yellow) Finland(green)	4	3
8	Russia (Kaliningrad) (orange)		Poland(red) Lithuania(green)	4	3
9	Lithuania (green)	Poland(red)	Belarus(yellow) Latvia(red)	10	3
10	Latvia(red)	Russia(orange)	Estonia(yellow)	8	3
11	Russia (orange)	Belarus(yellow)	Ukraine(green)	8	3
12	Russia (orange)		Georgia(yellow) Azerbaijan(green)	4	3
13	Kazakhstan (yellow)	Kyrgzstan(orange)	Uzbekistan(green)	8	3
14	Kashmir (yellow)	Afghanistan(orange) India(orange)	Pakistan(green)	13	3
15	Uzbekistan (green)	Kazakhstan(yellow) Afghanistan(orange)	Turkmenistan(red)	17	4
16	Georgia (yellow)	Azerbaijan (two parts,green)	Armenia(orange) Turkey(red)	10	4

17	Ukraine (green)	Poland(red)	Slovakia(yellow) Hungary(red) Romania(yellow) Moldova(red)	10	3
18	Finland (green)	Norway(yellow)	Sweden(red)	8	3
19	Poland (red)	Slovakia(yellow)	Germany(yellow) Czech(orange)	10	3
20	Germany (yellow)	Czech(orange)	Netherlands(green) Belgium(orange) France(green) Switzerland(orange) Austria(green)	12	3
21	Germany (yellow)		Denmark(green)	2	2
22	Hungary (red)	Austria(green) Romania(yellow)	Liechtenstein(yellow) Croatia(green) Yugoslavia(orange)	21	4
23	Yugoslavia (orange)	Romania(yellow)	Albania(green) Macedonia(yellow) Bulgaria(green)	12	3
24	France (green)	Switzerland(orange)	Italy(red)	8	3
25	France (green)		Monaco(yellow)	2	2
26	France (green)		Spain(orange)	2	2
27	Spain (orange)		Portugal(yellow)	2	2
28	Macedonia (yellow)	Albania(green) Bulgaria(green)	Greece(orange)	13	3
29	Turkey (red)	Azerbaijan(green)	Syria(yellow), Iraq(green) Iran(yellow)	12	3
30	Iraq (green)	Syria(yellow)	Jordan(red), Saudi Arabia(yellow) Kuwait(red)	12	3
31	Syria (yellow)		Lebanon(green)	2	2
32	Saudi Arabia (yellow)		Yemen(red),Oman(green), U.A.E.(red), Qatar(green)	4	3
33	Jordan (red)		Israel(orange)	2	2
34	Israel (orange)		Egypt(red)	2	2

35	Egypt (red)		Libya(orange) ,Sudan(yellow)	4	3
36	Libya (orange)	Sudan(yellow)	Tunisia(yellow) Algeria(green) Niger(yellow) Chad(green)	10	3
37	Algena (green)	Niger(yellow)	Morocco(yellow) Western Sahara(red) Mauritania(yellow) Mali(red)	10	3
38	Sudan (yellow)	Chad(green)	Central Africa(red) P.R.Congo(green) Uganda(red) Kenya(green) Ethiopia(red) Eritrea(green)	10	3
39	Ethiopia (red)	Eritrea(green) Kenya(green)	Djibouti(orange) Somalia(yellow)	14	4
40	Mali (red)	Mauritania(yellow) Niger(yellow)	Senegal(orange) Guinea(yellow) Cote d'Ivoire(orange) Burkina Faso(green)	14	4
41	Guinea (yellow)	Cote d'Ivoire (orange)	Sierra Leone(orange) Liberia(green)	10	3
42	Chad (green)	Niger(yellow) Central Africa(red)	Nigeria(red) Cameroons(yellow)	19	3
43	Niger (yellow)	Burkina Faso(green) Nigeria(red)	Benin(orange)	17	4
44	Burkina Faso (green)	Cote d'Ivoire (orange) Benin(orange)	Ghana(yellow), Togo(red)	14	4
45	Cameroons (yellow)	Central Africa(red)	Equatorial Guinea(orange) Gabon(red) Congo(orange)	12	3
46	P.R.Congo (green)	Uganda(red)	Angola(red) Zambia(yellow) Tanzania(red) Rwanda(yellow)	10	3
47	Zambia (yellow)	Angola(red) Tanzania(red)	Namibia(green) Botswana(orange) Zimbabwe(red) Mozambique(orange)	14	4
48	Botswana (orange)	Namibia(green) Zimbabwe(red)	South Africa(yellow)	17	4
49	Senegal (orange)		Gambia(yellow)	2	2

50	Guinea (yellow)	Senegal (orange)	Guinea-Bissau (green)	8	3
51	South Africa (yellow)	Mozambique (orange)	Swaziland(red)	8	3
52	Canada(red)		U.S.A.(orange)	2	2
53	U.S.A. (orange)		Mexico(yellow)	2	2
54	Mexico (yellow)		Guatemala(green) Belize(red)	4	3
55	Guatemala (green)		El Salvador(red) Honduras(orange)	4	3
56	Honduras (orange)		Nicaragua(yellow)	2	2
57	Nicaragua (yellow)		Costa Rica(red)	2	2
58	Costa Rica (red)		Panama(orange)	2	2
59	Panama (orange)		Colombia(yellow)	2	2
60	Colombia (yellow)		Ecuador(red), Peru(green) Brazil(red) Venezuela(green)	4	3
61	Brazil (red)	Venezuela(green)	Guyana(yellow) Surinam(green) French Guiana(yellow)	12	3
62	Brazil (red)	Peru(green)	Bolivia(yellow) Argentina(green) Uruguay(yellow)	12	3
63	Bolivia (yellow)	Argentina(green) Brazil(red)	Paraguay(orange)	17	4
64	Bolivia (yellow)	Peru(green) Argentina(green)	Chile(red)	13	3

Table 3 Colouring Records of China Map

No	Relative Center Region (Colour)	Coloured Adjacent Regions (Colour)	Uncoloured Adjacent Regions (Colour)	Type of Adjacent Region Section	Kinds Of Colouring
1	Beijing (red)		Tianjin(orange) Hebei(yellow)	3	3
2	Hebei (yellow)		Liaoning(green) Nei Mongol(red) Shanxi (green) Henan (red) Shandong (green)	6	3
3	Nei Mongol (red)	Liaoning (green)	Jilin (orange) Heilongjiang (green)	10	3
4	Henan (red)	Shanxi (green) Shandong (green)	Shensi(orange) Hubei(green) Anhui(orange)	15	3
5	Anhui (orange)	Hubei (green) Shandong (green)	Jiangxi(yellow) Zhejiang(green) Jiangsu(yellow)	15	3
6	Jiangxi (yellow)	Hubei (green) Zhejiang (green)	Hunan(red) Kwangtung(green) Fujian(red)	15	3
7	Jiangsu (yellow)	Zhejiang (green)	Shanghai(red)	8	3
8	Kwangtung (green)		Hong Kong (red)	2	2
9	Hunan (red)	Hubei (green) Kwangtung (green)	Chongching (yellow) Guizhou (green) Kwangsi (yellow)	15	3
10	Chongching (yellow)	Shensi (orange) Guizhou (green)	Sichuan(red)	17	4
11	Shensi (orange)	Nei Mongol (red) Sichuan (red)	Ningxia (green) Gansu (yellow)	14	4
12	Sichuan (red)	Gansu (yellow) Guizhou(green)	Qinghai (green) Xizang (yellow) Yunnan (orange)	21	4
13	Qinghai (green)	Gansu (yellow) Xizang (yellow)	Xinjiang (red)	13	3

EXPLANATION TO COLOURING OF WORLD MAP AND CHINA MAP

1. The base maps used for the colouring of world map and China map were traced according to the standard maps published by Xingqiu Cartographic Publishing House in April 1998, whose scales are $1:3300\times10^4$ and $1:600\times10^4$ respectively.

2. Basically, the process which the two maps were coloured was the same. At the time of beginning, one uncoloured region was optionally determined as a relative center region and coloured with one colour at will. The colour that the relative center region was coloured was the 1st one for the adjacent region section dependent of the relative center region. Number of adjacent region sections in the relative center region was studied, then these sections were coloured according to the type of every adjacent region section. After finishing the colouring of every adjacent region section dependent of the 1st relative center region, one in the above coloured adjacent regions was determined as the next relative center region, whose colour coloured was again used as the 1st one for the adjacent region section related to the relative center region. As mentioned above, number of adjacent region sections related to the relative center region was determined, and then these sections were coloured according to the type of every adjacent region section. According to this way, the relative center region was shifted, step by step, to make the later adjacent region section relate to the former, and developed gradually till all regions of the whole map were coloured completely. In the case of every part on map, by way of example (colouring of world map), if European, Asian and African countries are coloured at first, the following colouring should be South and North America countries; Otherwise, the adjacent region sections are divided and then coloured in the relative center region whose restarting determination is needed for the later adjacent region sections. In comparison with China map, some simple encircled regions were only added in the colouring of world map, in addition to more similar types of adjacent region sections and many increase in separate part.

3. One or two colour was optionally given for the separate parts respectively. Take world map for example: Japan (green), Philippines (orange), Singapore (green), Indonesia and Papua New Guinea (yellow, green), Sri Lanka (green), Bahrain (orange), Madagascar (yellow), Australia (red), New Zealand (orange), Greenland (yellow), Iceland (red), Britain and Ireland (yellow, green),Cuba (orange), Jamaica (yellow), Haiti and Dominican Rep. (red, yellow), Puerto Rico (orange), Trinidad And Tobago (red) and

so on. Say more than was not needed for other island countries in oceans; Take China map for example: Taiwan (green), Hainan (orange) and Macao (yellow).

4. The encircled countries were coloured according to the above principles: San Marino (yellow), Vatican City State (green), Luxembourg (red), Bosnia and Herzegovina (yellow), Mongolia (green), Nepal (orange), Sikkim (green), Bhutan (orange), Burundi (orange), Malawi (green) and Lesotho (green) etc.

5. With exception of separate parts and encircled regions, 63 ones in 64 adjacent region sections on world map were coloured according to the colouring kinds of every type of adjacent region sections; The 16th adjacent region section, of course, can be coloured according to the colouring kinds of the same adjacent region section (three colours); However, because Azerbaijan was divided into two parts, i.e. the eastern part and western part (exceptional case), if Turkey is coloured green, this will easily give people a false impression on confusing with the western part of Azerbaijan that was coloured green though they have no common boundary and are very close, so the colouring of Turkey was specially changed into red; One colour was added but their colouring was not more than four colours. Continentally, U.S.A and Russia fall into two parts, so the colouring of adjacent region sections where these two countries lie is not affected. 13 adjacent region sections on China map were coloured according to the colouring kinds of every type of adjacent region sections.

6. With exception of separate parts and encircled regions, all 64 adjacent region sections on world map fell into 12 adjacent region section types, while all 13 adjacent region sections on China map fell into 10 adjacent region section types. In the case of these two maps,12 adjacent region section types taking the shape of unclosed arrangement appeared, but 8 adjacent region section types (Type 1,5,7,9,11,16,18 and 20) taking the shape of closed arrangement did not emerge and Type 3 adjacent region section appeared once only on China map. All these showed that the actual adjacent region sections on maps, for the most part, take the shape of more unclosed arrangement than closed arrangment (only a few).

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费尔马猜想的初等数学方法的直接证明*

引言：本文在前人特殊证明的基础上,证明了两条新定理，运用初等数学方法，简单明了地直接证明了费尔马猜想一般地成立。

关键词：两条新定理　　费尔马猜想　　初等数学方法　　直接证明　　一般成立

一、费尔马猜想的提出

费尔马猜想又叫费尔马猜测、费尔马大定理、费尔马最后定理。这是数论著名的问题，被称为现代数学的三大难题之一，又被称为数学的“千古之谜”[1]。

大约在 1637 年，法国数学家费尔马（Pierre de Fermat），在一本《算术》书的空白处写了一段简短的笔记：“不可能把一个正整数的三次方幂分成两个三次方幂的和、一个四次方幂分成两个四次方幂的和，或者一般地，不可能把一个次数大于 2 的正整数方幂分成两个同方幂的和。”

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接着，他又写道："我发现了这个论断的证明，但是书上的空白太窄了，写不下。"上述命题用不定方程来表述，就是说：

设 $x>0$，$y>0$，$z>0$，（x，y，z）=1，正整数 $n>2$，

不定方程

$$x^n+y^n=z^n \quad \cdots\cdots\cdots\cdots \quad (1)$$

无正整数解。

费尔马的上述笔记，在他死后五年的 1670 年才由他的儿子发表。人们查遍了费尔马的笔记和书信，发现费尔马只证明了 $n=4$ 时（1）式无正整数解（下文称为"费尔马证明"），而没有一般地证明（1）式无正整数解。这就是费尔马猜想。

二、前人的证明成果

自从费尔马猜想发表以后，三百多年来，世界上许多优秀的数学家，为了证明它，想尽办法，奋斗不息。因为大于 2 的 n 可以表述为大于 2 的素数 P 和 4 以及它们的倍数，所以除了"费尔马证明"以外，人们把（1）式改写成不定方程

$$x^P+y^P=z^P \quad \cdots\cdots\cdots\cdots \quad (2)$$

只要证明（2）式无正整数解，即（1）式无正整数解。1753 年，瑞士数学家欧勒（Leonhard Euler）证明了 $P=3$ 时（2）式无正整数解（下文称

为“欧勒证明”)。1825 年，法国数学家勒让德（Adrien Marie Legendre）和德国数学家狄利克雷（Peter Gustar Lejeune Dirichlet）分别证明了 P=5 时（2）式无正整数解（下文称为“勒让德—狄利克雷证明”)。1839 年，法国数学家拉梅（Grabriel Lame）证明了 P=7 时（2）式无正整数解（下文称为“拉梅证明”)。前人用直接证明的方法，仅仅取得了上述四个特殊证明成果。他们还没有得到费尔马猜想的一般的直接证明。1844 年，德国数学家库麦尔（Ernst Eduard Kummer）引入“理想素因子”概念，通过许多数学家的继续努力，创立了代数数论，费尔马猜想的间接证明不断取得了突破。1975 年，约翰逊（W.Johnson）通过电子计算机，证明了 2<P<30’000 时（2）式无正整数解。1977 年，瓦格斯塔夫（Samuel S.Wagstaff）更在大型计算机的帮助下，证明了 2<P<125’000 时（2）式无正整数解。罗寒进一步证明了 2<P<4’100 万时（2）式无正整数解。有人甚至证明了在 x、y、z 与 P 互素时，2<P<253’749’889（这在无穷多的正整数中只是微不足道的一小部份）时（2）式无正整数解**。直至 1995 年，英国数学家安德鲁・维尔斯用代数几何法间接证明了（2）式一般无正整数解。可是，2001 年 11 月出版的《数理化之谜》一书根本就没有提及安德鲁·维尔斯的证明，还是明确指出“费尔马大定理仍然是一个猜测。” [*1*]

**科学技术百科全书，第一卷•数学(科学出版社，北京，1980)。

三、 本文作者的证明

本文作者将在前人特殊证明成果的基础上，运用初等数学方法，直接证明费尔马猜想一般地成立。

为了证明（1）式无正整数解，首先证明两条新的定理。

定理 1. 设正整数 $K>1$，则任意一个正整数 N 开 K 次方的算术根，不是正整数就是正无理数。

证明：分两步进行。

第一步：设正整数 $A>0, B>0, m>0$, A/B 为有理数，$N^{1/K}=A/B$，如果 B 整除 A，$A/B=m$，则 $N=(A/B)^K=m^K$ 成立，证明有的正整数开 K 次方的算术根仍为正整数。实际上，由于素数的个数无穷，任何由素数或素数因子乘积的 K 次方幂构成的正整数，它开 K 次方的算术根必然为正整数，所以有无穷多个正整数开 K 次方的算术根仍为正整数。

如果 B 不整除 A，则 $N^{1/k}=A/B$，即 $N=(A/B)^K$，而 A/B 为分数，$(A/B)^K$ 总是分数，这样 N 就只能是一个分数而不能成为一个正整数了，演算结果与已知条件矛盾，$N^{1/k}=A/B$ 不能成立，$N^{1/k}$ 不能为分数。

第二步：设 W 为正无理数，$N^{1/k}=W$，则 $N=W^K$，因为 W 是正整数 N 开 K 次方的算术根的正无理数，所以 $N=W^K$ 成立，证明有的正整数开 K 次方的算术根为正无理数。实际上，任何由素数或素数因子乘积的非 K 次方幂构成的正整数，它开 K 次方的算术根则为正无理数，因而也有无穷多

个正整数开 K 次方的算术根为正无理数。

综上所述，任意一个正整数 N 开 K 次方的算术根，不是正整数就是正无理数。定理 1 证明完毕。

定理 2. 若 $x>0, y>0, z>0, (x, y, z)=1$, 正整数 $n>2, K>1$, 则不定方程

$$x^n+y^n=z^n$$

无正整数 K 次方根的正无理数解。

证明：假设上式[即（1）式]有正整数 K 次方根的正无理数解，C、D、E 为正整数，（C，D，E）=1，$C^{1/k}$、$D^{1/k}$、 $E^{1/k}$ 都不是正整数而是正无理数,$x=C^{1/k}$, $y=D^{1/k}$, $z=E^{1/k}$, 则（1）式可以写成

$$(C^{1/k})^n+(D^{1/k})^n=(E^{1/k})^n \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

设正整数 $j>0$，则当 $n=3jk$ (当 $j=1$，$3jK=3K$)时，（3）式可以写成

$$(C^{1/k})^{3jk}+(D^{1/k})^{3jK}=(E^{1/k})^{3jK}$$

即 $$\{[(C^{1/k})^K]^j\}^3+\{[(D^{1/k})^K]^j\}^3=\{[(E^{1/k})^K]^j\}^3 \quad \ldots\ldots (4)$$

(4)式中,显然$(C^{1/k})^K$ 、$(D^{1/k})^K$ 、$(E^{1/k})^K$ 为正整数，根据乘法性质，$[(C^{1/k})^k]^j$、$[(D^{1/k})^k]^j$、$[(E^{1/k})^k]^j$都是无穷多个正整数，(4) 式变成了（1）式在 n=3 时有无穷多组正整数解。这与“欧勒证明”矛盾，假设不能成立，即（3）式不能成立，(1) 式不能有正整数 K 次方根的正无理数解。定理 2 证明完毕。

下面证明（1）式无正整数解。

（1）式可以改写成

$$(x^{n/3})^3+(y^{n/3})^3=(z^{n/3})^3 \quad \cdots\cdots\cdots\cdots \quad (5)$$

只要证明不定方程（5）无正整数解，即不定方程（1）无正整数解便得到了证明。

假设（5）式有正整数解，即 x、y、z 为正整数，则根据乘法性质，x^n、y^n、z^n 也是正整数。根据定理 1，$x^{n/3}$、 $y^{n/3}$、$z^{n/3}$ 不是正整数就是正无理数。它们如果是正整数，则与“欧勒证明”矛盾；如果是正无理数，则与定理 2 矛盾。因此，上述假设不能成立，（5）式无正整数解，不定方程（1）无正整数解得到了证明，费尔马猜想得到了一般的直接证明。

四、结 论

本文在“欧勒证明”的基础上，证明了两条新的定理，运用初等数学方法，简单明了地直接证明了费尔马猜想一般地成立。

同样，以定理 1 为前提，分别在“费尔马证明”、“勒让德—狄利克雷证明”或者“拉梅证明”的基础上，运用类似上述的证明方法，也可以证明定理 2，从而运用初等数学方法直接证明费尔马猜想一般地成立。因此，费尔马猜想这一类型的证明至少有四种。这一事实充分说明，费尔马猜想的初等数学方法的直接证明有多种，本文只是作者已知的最简单明了的一种证明。

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THE DIRECT PROOF OF FERMAT CONJECTURE BY METHOD OF ELEMENTARY MATHEMATICS*

INTRODUTION: On the basis of the predecessors' special proof, the author has proved two new theorems, and used the method of elementary mathematics to give simple, clear and direct proof of Fermat conjecture to be generally tenable.

KEY WORDS: Two new theorems, Fermat conjecture, Elementary mathematical method, Direct proof, Tenable generally.

1 PRESENTATION OF FERMAT CONJECTURE

Fermat conjecture is called as Fermat guess, Fermat's Greatest Theorem and Fermat's Last Theorem, and it is a well-known problem of number theory, which is called as one of three major problems in modern mathematics and as a forever mystery of mathematics.[1]

In about 1637, Pierre de Fermat, a mathematician in France, wrote a brief note on the margin in *Arithmetic*: " It is impossible to divide the third power of one positive integer into the sum of two the third powers, one the fourth power into the sum of two the fourth powers, or generally, the power greater than 2 of one positive integer into the sum of two the same powers." And he added that he found the proof of this inference, but there was no room to write it down on the margin in this book. The proposition stated above is expressed as indefinite equation, viz if $X>0$, $Y>0$, $Z>0$, $(X, Y, Z) = 1$, positive integer $n>2$, then indefinite equation

$$X^n + Y^n = Z^n \tag{1}$$

has no solution in positive integer.

In 1670 when Fermat died for 5 years, his son published his note mentioned above. A thorough search for notes and letters written by Fermat was made. It was found that Equation (1) has no solution in positive integer when Fermat proved $n = 4$ only, which is called as Fermat's proof below. However, Fermat did not generally proved that Equation (1) has no solution in positive integer. This is Fermat conjecture.

2 ACHIEVEMENTS IN PREDECESSORS' PROOF

It is over 300 years since Fermat conjecture was published. In order to prove Fermat conjecture, world many excellent mathematicians made every effort to do it unremittingly. Because n greater than 2 can be expressed as prime number P greater than 2, and 4 and their multiple as well, Equation (1) was rewritten as indefinite equation, viz

$$X^P + Y^P = Z^P \tag{2}$$

with the exception of Fermat's proof. So long as Equation (2) is proved to have no solution of positive integer, Equation (1) has no solution of positive integer. In 1753, as Leonhard Euler, a mathematician

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in Switzerland, proved P=3, Equation (2) has no solution of positive integer, which is called as Euler's proof below. In 1825, as Adrien Marie Legendre, a mathematician in France, and Peter Gustar Lejeune Dirichlet, a mathematician in Germany, proved P = 5 respectively, Equation (2) has no solution of positive integer, which is called as Legendre – Dirichlet's proof below. In 1839, as Grabriel Lame, a mathematician in France, proved P = 7 , Equation (2) has no solution of positive integer, which is called as Lame's proof below. By use of the method of direct proof, the predecessors obtained only four achievements in special proof stated above. They have not obtain the general direct proof of Fermat conjecture still. In 1844, Ernst Eduard Kummer, a mathematician in Germany, introduced the concept of ideal prime factor. With a unremitting effort made by many mathematicians, the algebraic theory of numbers was established, and a succession of breakthrough in indirect proof of Fermat conjecture was made. In 1975, W. Johnson proved Equation (2) has no solution of positive integer as 2<P<30000, with the help of computer. In 1977, Samuel S.Wagstaff, with the help of Large-scale computer, proved Equation (2) has no solution of positive integer as 2<P<125000. Furthermore, Rosser proved Equation (2) has no solution of positive integer as 2<P<41000000. A somebody proved Equation (2) has no solution of positive integer as X, Y and Z are coprime to P respectively, i.e. 2<P<253749889 (only a trivial part in infinitely positive integers).** Up to 1995, Andrew Wills, a mathematician in U.K., used the method in algebraic geometry to indirectly prove Equation (2) has no solution of positive integer generally. However, Andrew Wills'proof was not covered at all in Mysteries to Mathematics, Physics and Chemistry (Changyi et al, 2001), in which it was definitely noted that Fermat's Greatest Theorem remains to be a guess.[1]

3 THE AUTHOR'S PROOF

On the basis of achievements in predecessors' special proof, the author has used the method of elementary mathematics to directly prove Fermat conjecture to be generally tenable.

In order to prove Equation (1) has no solution of positive integer, two new theorems have been proved at first.

THEOEREM 1: If positive integer K>1, then the Kth positive root of an optional positive integer N is either positive integer or positive irrational number.

PROOF: The work is done in two steps.

STEP 1: Suppose positive integers A>0, B>0, m>0, A/B are rational numbers, then $N^{1/K}$ = A/B; if B is divided exactly by A, viz A/B=m, then $N=(A/B)^K=m^K$ is tenable. It is proved that the Kth positive root of some positive integers remains to be positive integer. In fact, the Kth positive root of any positive integer, which is composed of the Kth power of product of prime numbers or prime factors, is positive integer inevitably, due to infinitely prime numbers. So, it is infinite that the Kth positive root of positive integer remains to be positive integer.

If B is not divided exactly by A, then $N^{1/K}$=A/B, viz $N=(A/B)^K$, in which A/B is fractional number, and $(A/B)^K$ is always fractional number as well. In this way, N can but be a fractional

**An Encyclopedia of Science and Technology, Vol.1: Mathematics (The Science Publishing House, Beijing: 1980)

number, rather than a positive integer. The result of calculation is contrary to the known condition, so $N^{1/K}=A/B$ can not be tenable, and $N^{1/K}$ can not be fractional number.

STEP 2: Suppose W is positive irrational number, $N^{1/K}=W$, then $N=W^K$. Because W is positive irrational number of the Kth positive root of positive integer N, so $N=W^K$ is tenable. The Kth positive root of some positive integers is proved to be positive irrational number. In fact, the Kth positive root of any positive integer, which is composed of the non-Kth power of product of
prime numbers or prime factors, is positive irrational number. So it is infinite that the Kth positive root of positive integer is positive irrational number also.

In one word, the Kth positive root of an optional positive integer N is either positive integer or positive irrational number. The proof of Theorem 1 is finished.

THEOREM 2: If $X>0$, $Y>0$, $Z>0$, $(X,Y,Z)=1$, positive integer $n>2$, $K>1$, then Indefinite Equation $X^n + Y^n = Z^n$ has no solution of positive irrational number of the Kth root of positive integer.

PROOF: If Equation (1) has solution of positive irrational number of the Kth root of positive integer, C, D and E are positive integer, (C, D, E)=1, $C^{1/K}$, $D^{1/K}$ and $E^{1/K}$ are not positive integer but positive irrational number, $X= C^{1/K}$, $Y= D^{1/K}$, $Z= E^{1/K}$, then Equation (1) can be expressed as

$$(C^{1/K})^n + (D^{1/K})^n = (E^{1/K})^n \qquad (3)$$

If positive integer $j>0$, then when $n=3jk$(when $j=1$, $3jk=3k$), Equation (3) can be expressed as

$$(C^{1/K})^{3jk} + (D^{1/K})^{3jk} = (E^{1/K})^{3jk}$$

viz $$\{[(C^{1/K})^K]^j\}^3 + \{[(D^{1/K})^K]^j\}^3 = \{[(E^{1/K})^K]^j\}^3 \qquad (4)$$

In Equation (4), it is obvious that $(C^{1/K})^K$, $(D^{1/K})^K$ and $(E^{1/K})^K$ are positive integers. According to property of multiplication, $[(C^{1/K})^K]^j$, $[(D^{1/K})^K]^j$, and $[(E^{1/K})^K]^j$ are infinitely positive integers, and Equation (4) becomes that Equation (1) has infinite solution of positive integers when n=3. This is in contradiction with Euler's proof, so the assumption can not be tenable, viz Equation (3) can not be tenable. Equation (1) can not have solution of positive irrational number of the Kth root of positive integer. The proof of Theorem 2 is finished.

The following is to prove Equation (1) has no solution of positive integer. Equation (1) can be rewritten as

$$(X^{n/3})^3 + (Y^{n/3})^3 = (Z^{n/3})^3 \qquad (5)$$

So long as Indefinite Equation (5) is proved not to have solution of positive integer, it is proved that Indefinite Equation (1) has no solution of positive integer.

If Equation (5) has solution of positive integer, viz X, Y and Z are positive integers, then X^n, Y^n and Z^n are positive integers also, according to property of multiplication. In terms of Theorem 1, $X^{n/3}$, $Y^{n/3}$ and $Z^{n/3}$ are either positive integer or positive irrational number; If they are positive integer, this is contrary to Euler's proof; If they are positive irrational number, this is contrary to Theorem 2. Thus, the above assumption can not be tenable, Equation (5) has no solution of positive integer, and it is proved that Indefinite Equation (1) has no solution of positive integer. In this way, the general direct proof of Fermat conjecture is obtained.

4 CONCLUSIONS

Based on Euler's proof, two new theorems have been proved. By use of elementary mathematics method, the simple, clear and direct proof of Fermat conjecture has been given to be generally tenable.

Similarly, taking Theorem 1 as precondition, based respectively on Fermat's proof, Legendre-Dirichlet's proof or Lame's proof, the similar proof methods above can be used to prove Theorem 2, so as to use the method of elementary mathematics for proving Fermat conjecture to be generally tenable. Obviously, this type of the proof of Fermat conjecture has four kinds at least. This fact has fully showed that the direct proof of Fermat conjecture by the method of elementary mathematics has several kinds, but this paper is a kind of the proof known, by the author, to be the simplest and clearest.

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冰雹猜想的证明*

内容提要：本文揭示了大于 1 的自然数的本质特征，重新定义了自然数，对自然数进行了新的分类：纯粹 2^n 类的自然数和非纯粹 2^n 类的自然数，证明了一条新的定理，找出了冰雹猜想的本质规律，一般地证明了该猜想对于所有自然数都成立。

关键词：自然数的本质特征　新定义　新分类　新定理 冰雹猜想 一般证明

一、序 言

“冰雹猜想”是 20 世纪世界著名的数学问题。它是指任意一个自然数，如果它是偶数，就用 2 去除它；如果它是奇数，将它乘以 3 之后再加 1，这样反复演算，最终必然得 1。

这个问题最初由谁提出来，他（她）是什么人，是在什么时候、什么场合提出来的，都弄不清楚。不过它似乎并不古老。20 世纪 30 年代，德国汉堡大学的学生考拉兹就研究过它。1952 年一位英国的数学家发现了它。几年之后，它又被一位美国数学家发现。人们在大量的演算中发现，算出来的数字忽大忽小，有的演算过程很长。有人把演算过程形容为云中的小水滴,在高空气流的作用下，忽高忽低，遇冷结冰，体积越来越大，最后变成冰雹落了下来，而演算的数字最后也像冰雹一样掉了下来，变成了 1。人们因而给上述问题起了一个形象的名字——冰雹猜想。这个问题最早由角谷静夫介绍到日本，所以日本人又称它为“角谷猜想”。[1]、[2]

这个猜想自 20 世纪 50 年代起，一再引起了人们的广泛兴趣。1960 年日本数学家角谷静夫初次听到这个猜想，他说：“有一个月，耶鲁大学每一个人都在研究这个

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问题，但没有任何结果。我到芝加哥大学提出这个问题之后，也出现了同样的现象。”[2]一位美国数学家也说：“有一个时期，在美国的大学里，它几乎成了最热门的话题，数学系和计算机系的大学生，差不多人人都在研究它。”[1]人们一遍又一遍地从 1 开始对许多自然数进行演算，结果都说明冰雹猜想成立。日本东京大学的米田信夫运用计算机，对从 1 到 2^{40}（大约相当于 1.2 万亿[2]，一说到 10995 亿 1162 万 7776[1]）之间的所有自然数逐一进行了演算，说明冰雹猜想都是正确的。虽然人们对大量的自然数进行了演算，但是自然数有无穷多个，逐一进行演算是演算不完的，“大量”并不能代替“全体”，演算的自然数再多，也代替不了数学证明，况且演算的自然数再多，与无穷多的全部自然数相比，比值还是相当于 0。因此，冰雹猜想的证明，在此之前还是没有解决的一个谜。本文的目的在于从自然数的本质特征出发，重新定义自然数，对自然数进行新的分类，找出冰雹猜想的本质规律，从而一般地证明该猜想对于所有自然数都成立。

二、证　明

任何一个大于 1 的自然数的本质特征就是都由一定的素数因子构成。从自然数的本质特征出发，　自然数可以重新定义如下：自然数就是 1 和所有素数以及两个或两个以上的素数因子各种组合的数的总和。自然数 N 可以表达为 $N=2^n\ (2m+1)$（$m\geqslant0,n\geqslant0$,整数）。当 $m=0$，$n=0$ 时，则 $N=1$；当 $m=0$，　$n>0$ 时，则 $N=2^n$，为纯粹 2^n 类自然数；当 $m>0$，$n>0$ 时,则 $N=2^n\ (2m+1)$，为非纯粹 2^n 类自然数的偶数；当 $m>0$ ，$n=0$ 时，则 $N=2m+1$，为非纯粹 2^n 类自然数的奇数。因此，根据自然数本质特征的不同，自然数可以重新分为两类：第一类为纯粹 2^n 类的自然数（$n\geqslant0$），即 1（仅把 1 看成 $1=2^0$）和一个或一个以上素数因子为 2 的数；第二类为非纯粹 2^n 类的自然数，即自 然 数
$2^n\ (2m+1)$（$m>0$，$n\geqslant0$），包括只有一个奇素数因子的数和两个或两个以上素数因子而至少有一个奇素数因子的数。运用这种自然数新的分类，就可以证明冰雹猜想了。

冰雹猜想中的“任意一个自然数”，当然不是偶数就是奇数。偶数包括纯粹 2^n 类的自然数和非纯粹 2^n 类自然数的偶数。“如果它是偶数，就用 2 去除它”，实质上就是要减少、直至消除该自然数中的素数 2 因子。如果它是纯粹 2^n 类的自然数（它与所有非纯粹 2^n 类的自然数无关，对所有非纯粹 2^n 类的自然数都可以不管），则用 2 去除 n 次，最终得数为 $2^0=1$。就纯粹 2^n 类的自然数而言，冰雹猜想得到了证明。例如 $16384=2^{14}$，用 2 去除 14 次，最后得数为 $2^0=1$。如果初始给定的那个自然数，除了有 n 个素数 2 因子以外，还有一个或一个以上的奇素数因子，则用 2 去除该自然数 n 次以后，剩下的就是一个奇素数或一个以上的奇素数的乘积，它必然是一个奇数 $2m+1$（$m>0$），也是一个非纯粹 2^n 类自然数的奇数。例如 $12=2^2\times3$,用 2 去除两次以后，剩下就只有奇素数 3 了；$280=2^3\times5\times7$，用 2 去除三次以后，剩下的数就是 $5\times7=35$。这个在演算过程中产生的奇数，与初始给定的奇数一样对待。

对于奇数，为了证明冰雹猜想是否成立，首先需要考察奇数 $2m+1$“乘以 3 之后再加 1”是否能够变成纯粹 2^n 类的自然数，即证明不定方程

$$3\times(2m+1)+1=2^n \cdots\cdots (1)$$

是否能够成立。

（1）式可以改写成

$$6m+3+1=2^n$$

即 $3m+2=2^{n-1}$

显然上式的 m 为偶数，设 m = 2D, D>0，为正整数，则

$3D+1=2^{n-2}$，D 只能是奇数。

$$D=(2^{n-2}-1)/3 \cdots\cdots (2)$$

为了使（2）式有正整数解，n-2 必须为偶数。设 n-2=2s, s>0.为正整数，则

$$D=[(2^2)^s-1]/3$$

$$D=(4-1)(4^{s-1}+4^{s-2}+\cdots\cdots+4^2+4+1)/3$$

$$D=4^{s-1}+4^{s-2}+\cdots\cdots+4^2+4+1 \cdots\cdots (3)$$

在（3）式中，与每一个 S 值对应，都有一个奇数 D：

S=1，2，3，4，5……

D=1，5，21，85，341……

不定方程（3）成立，有无穷多组解，即不定方程（1）成立，有无穷多组解。这说明在正整数集合中，有无穷多个非纯粹 2^n 类自然数的奇数 D“乘以 3 之后再加 1”，都可以变成纯粹 2^n 类的自然数。

那么，任意一个奇数 2m+1（m>0），按照冰雹猜想所说的演算方法反复演算下去，是不是都可以变成奇数 D 呢？

所谓奇数就是只有一个或一个以上奇素数因子的数 2m+1（m>0）。“如果它是奇数，将它乘以 3 之后再加 1”，实质上就是要改变构成该非纯粹 2^n 类自然数的奇素数因子，因为“将它乘以 3 之后再加 1”所构成的新的自然数，既不能被原自然数中所有的素数因子整除，也不能被奇素数 3 整除，所以新的自然数既不含原自然数的奇素数因子，也不含奇素数 3 因子。例如 385=5×7×11，385×3+1=1156，$1156=2^2\times17^2$，1156 既不含原自然数的奇素数因子 5、7、11，也不含奇素数 3 因子。这样，就把由原有的奇素数因子构成的那个非纯粹 2^n 类的自然数排除掉了，构成了一个新的自然数。因为任意一个自然数，按照冰雹猜想所说的演算方法，除了纯粹 2^n 类的自然数 4—2—1 之间有循环关系以外（这个循环关系在第一次演算到得数为 1 以后就可以不管了，所以不必考虑），其它纯粹 2^n 类的自然数和所有非纯粹 2^n 类的自然数之间都没有循环关系。这种关系有下列定理证明。

定理：任意一个大于 1 的自然数，如果它是偶数，就用 2 去除它；如果它是奇数，将它乘以 3 之后再加 1，结果都没有循环关系。

证明：当任意一个大于 1 的自然数是纯粹 2^n 类的自然数时（n>0），用 2 去除 n 次，商数按照公比为 1/2 的等比级数减小，最后结果为 1，显然没有循环关系。

当任意一个大于 1 的自然数是非纯粹 2^n 类自然数的偶数 $2^n(2m+1)$(m>0 ,n>0)时,在用 2 去除 n 次的过程中，商数也是按照公比为 1/2 的等比级数减小，最后结果为 2m+1，也没有循环关系。

当任意一个大于 1 的自然数是非纯粹 2^n 类自然数的奇数 2m+1（m>0）时，假设它在演算过程中有循环关系，因为 2m+1 是奇数，而（2m+1）×3+1 是偶数，所

以必须满足

$$(2m+1)\times 3+1=2^n(2m+1)$$
$$3(2m+1)-2^n(2m+1)=-1$$
$$(3-2^n)(2m+1)=-1$$

上式中显然 $2m+1\geqslant 3$,当 $n=1$ 时,$3-2^n=1$，则

$$(3-2^n)(2m+1)\geqslant 3$$

即 $$(3-2^n)(2m+1)\neq -1$$

当 $n>1$ 时，$3-2^n\leqslant -1$,则

$$(3-2^n)(2m+1)\leqslant -3$$

同样 $$(3-2^n)(2m+1)\neq -1$$

上述假设不能成立，$(2m+1)\times 3+1$ 没有循环关系。定理证明完毕。

根据上述定理，任意一个自然数按照冰雹猜想所说的演算方法，每一步演算都排除掉一个有关的非纯粹 2^n 类的自然数。因为任意一个非纯粹 2^n 类的自然数,按照冰雹猜想所说的方法演算时,它与其它非纯粹 2^n 类的自然数之间的关系各不相同,有的根本没有关系,有的关系较少, 有的关系较多,所以对没有关系的非纯粹 2^n 类的自然数,可以置之不理;而对有关系的非纯粹 2^n 类的自然数,则“这样反复演算”,有多少就排除多少。又因为非纯粹 2^n 类自然数的奇数 D 在自然数集合中有无穷多个,所以任意一个非纯粹 2^n 类的自然数,在改变奇素数因子而构成新的自然数时,就随时都有可能变成非纯粹 2^n 类自然数的奇数 D。例如 21 就是非纯粹 2^n 类自然数的奇数 D,只演算一步就变成了纯粹 2^n 类的自然数：$21\times 3+1=64$。有的非纯粹 2^n 类的自然数,经过较长的演算过程,才能变成非纯粹 2^n 类自然数的奇数 D,例如 27 要经过 106 步演算,才变成非纯粹 2^n 类自然数的奇数 D，然后再演算一步便变成了纯粹 2^n 类的自然数 16。有的一定的充分大的非纯粹 2^n 类自然数,从理论上说,纵使在很长的演算过程中, 还没有变成非纯粹 2^n 类自然数的奇数 D,但是,在自然数集合中的一定区间内,与该自然数有关的非纯粹 2^n 类自然数的个数总是有限的,按照冰雹猜想所说的“这样反复演算”下去,即把排除有关的非纯粹 2^n 类自然数的演算继续进行下去,把有关的非纯粹 2^n 类的自然数一个接一个地排除掉,最后与初始给定的那个非纯粹 2^n 类自然数有关的自然数,便只剩下非纯粹 2^n 类自然数的奇数 D 了,当它再一次改变前一个非纯粹 2^n 类自然数的奇素数因子而构成新的自然数时,便只能变成非纯粹 2^n 类自然数的奇数 D 了，然后再演算一步便变成了纯粹 2^n 类的自然数。初始给定的任意一个非纯粹 2^n 类的自然数,通过“这样反复演算”,都必然变成一个纯粹 2^n 类的自然数,然后再用 2 去除 n 次,正如冰雹猜想所说的“最终必然得 1”。冰雹猜想对于初始为任意一个非纯粹 2^n 类的自然数也同样得到了证明。冰雹猜想完全得到了证明。

三、结 论

大于 1 的自然数的本质特征就是都由一定的素数因子构成。从自然数的本质特征出发，自然数可以重新定义为：自然数就是 1 和所有素数以及两个或两个以上素数因子各种组合的数的总和。根据自然数本质特征的不同，自然数可以重新分为两类：第一类为纯粹 2^n 类的自然数，即 1（$2^0=1$）和有一个或一个以上素数因子为 2 的数；第二类为非纯粹 2^n 类的自然数，即自然数 2^n（2m+1）（m>0，n≥0），包括只有一个奇素数因子的数和两个或两个以上素数因子而至少有一个奇素数因子的数，也即非纯粹 2^n 类自然数的偶数和奇数。运用自然数新的分类和新的定理，找出了冰雹猜想的本质规律，指出冰雹猜想中的“任意一个自然数”，当然不是偶数就是奇数，不是纯粹 2^n 类的自然数就是非纯粹 2^n 类的自然数。“如果它是偶数，就用 2 去除它”，实质上就是要减少、直至消除该自然数中的素数 2 因子。如果它是纯粹 2^n 类的自然数，则用 2 去除 n 次，最终得数为 1。就纯粹 2^n 类的自然数而言，冰雹猜想得到了证明。“如果它是奇数，将它乘以 3 之后再加 1”，实质上就是要改变构成该自然数的奇素数因子，而把由原有的奇素数因子构成的那个非纯粹 2^n 类的自然数排除掉。任意一个非纯粹 2^n 类的自然数，通过“这样反复演算”，都必然变成一个纯粹 2^n 类的自然数，然后再用 2 去除 n 次，则“最终必然得 1”。冰雹猜想对于任意一个非纯粹 2^n 类的自然数也同样得到了证明。冰雹猜想完全得到了证明。这就一般地证明了冰雹猜想对于所有自然数都成立。

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THE PROOF OF "HAIL" CONJECTURE *

ABSTRACT: An essential characteristic of natural number greater than 1 is discovered. Natural numbers are redefined and reclassified: pure 2^n class natural numbers and non-pure 2^n class natural numbers. One new theorem has been proved and some essential laws of " hail" conjecture have been discovered. It generally has been proved that this conjecture is tenable for all natural numbers.

Key words: Natural number, Essential characteristic, New definition, New classification, New theorem, "Hail" conjecture, Prove generally.

1 INTRODUCTION

"Hail" conjecture is one well-known mathematical problem in the world in the 20^{th} century. It means that one optional natural number, if even number, is divided by 2; if odd number, multiplied by 3 and then added to 1, after that divided by 2 again. This calculating process goes on further in such a way. The final result equals 1 inevitably.

Down to date, it is not clear that this problem was said initially by who in which place at what time. However, it seems not to be age-old. In 1930s, Kaulatz, who was a student in Hamburg Univ, made a study on it. In 1952, one mathematician in Britain met with it. Through the years later, one mathematician in USA met with it also. In the process of lots of calculation, it was found that the calculated number was great and small by turns, and some calculating processes were much longer. Someone described this calculating process as a small drop in cloud, which, by the action of air current in the high sky, is high and low by turns, freezing at cold temperature, becoming bigger and bigger, growing into a hailstone and at last falling down from the sky. Also, the number to be calculated is the same that the said hail falls down finally, and becomes 1. The problem mentioned above, therefore, was name as "hail" conjecture formally. The first one who introduced "hail" conjecture to Japan was Kakutani Shizuo, so the Japanese people called it as "Kakutani conjecture" [1~2].

Since 1950s, "hail" conjecture has held many people's interest. Kakutani Shizuo, a mathematician in Japan, learnt this conjecture for the first time in 1960. He said: " In a certain month, everyone in Yale Univ made a study on this conjecture, but no result was obtained. After I presented it in Chicago Univ, the same phenomenon took place also." [2] And one mathematician in USA said:

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" In a certain period, this conjecture almost became the hottest topic of conversation in USA universities, and the most of students in the mathematics and computer department researched into it." [1] The calculation of many natural numbers beginning from 1 was made over and over again. All results showed that "hail" conjecture is tenable. Yoneda Nobuo, who worked in Tokyo Univ in Japan, used a computer to make one-by-one calculation of all natural numbers from 1 to 2^{40} (about 1200 billion [2], i.e. 1099511627776[1]), and his result showed that "hail" conjecture is tenable also. Though a lot of natural numbers were calculated, natural numbers are infinite. By performing one-by-one calculation, it is not ended that natural numbers are calculated, "Lot" can not replace "total". Even if the calculated natural numbers are much more, the mathematical proof can't be replaced by this calculation. Moreover, even if the calculated natural numbers are much more, the ratio corresponds to 0, compared with infinite natural numbers. The proof of "hail" conjecture, therefore, remains a mystery not to be solved heretofore. The aims of this paper are to redefine natural numbers, according to the essential characteristic of natural number, reclassify natural numbers, discover some essential laws of "hail" conjecture and then generally prove that this conjecture is tenable for all natural numbers.

2 PROOF OF "HAIL" CONJECTURE

An essential characteristic of any optional natural number greater than 1 means that this natural number is composed of prime number divisors which are given. According to the essential characteristic of natural number, natural numbers can be redefined: Natural number is the sum of numbers, which 1 and combined with all prime numbers and two or more than two prime number divisors. The natural number N Can be expressed as $N=2^n(2m+1)$ ($m\geqslant 0, n\geqslant 0$, integer). If $m=0, n=0$, then $N=1$; If $m=0, n>0$, then $N=2^n$, which is pure 2^n class natural numbers; If $m>0$, $n>0$, then $N=2^n(2m+1)$, which is even numbers of non-pure 2^n Class natural number; If $m>0, n=0$, then $N=2m+1$, which is odd numbers of non-pure 2^n Class natural number. According to the difference in the essential characteristic of natural number, natural numbers can be reclassified into two classes: The first class is pure 2^n class natural numbers ($n\geqslant 0$), i.e. 1(1 is regarded as $1=2^0$ only) and the number which one or more than one prime number divisors are 2; The second class is non-pure 2^n class natural numbers, i.e. natural numbers $2^n(2m+1)$ ($m>0, n\geqslant 0$), including the number which has only one odd prime number divisor and the number which has two or more than two prime number divisors in which there is one odd prime number divisor at least. By use of the new classification of natural numbers, "hail" conjecture can be proved.

"An optional natural number" in "hail" conjecture, of course, is either even number or odd number. Even numbers include pure 2^n class natural numbers and even numbers of non-pure 2^n class natural numbers. "If it is even number, it is divided by 2." In fact, prime number 2 divisors in this natural number are to be reduced, till they are eliminated. If it is pure 2^n class natural number, which is independent of all non-pure 2^n class natural numbers and need not be taken into consideration of them, it is divided by 2 for n times, and the final result is $2^0=1$. As far as pure 2^n class natural numbers are concerned, "hail" conjecture has been proved. For example: $16384=2^{14}$, that is to say, 16384 is divided by 2 for 14 times; Thus, its result is $2^0=1$. If the initially-given natural number, besides n prime number

2 divisors, has one or more than one odd prime number divisors, it is divided by 2 for n times, then the remainder is one odd prime number or the product of more than one odd prime numbers, which is one odd number 2m+1(m>0) inevitably and one odd number of non-pure 2^n class natural number also. For example: $12=2^2\times3$, divided by 2 for twice, the remainder is odd prime number 3 only, $280=2^3\times5\times7$,divided by 2 for thrice, the remainder is $5\times7=35$. The processing of odd number from this calculating process is the same as that of the initially given odd number.

As far as odd number concerned, in order to prove that "hail"conjecture is tenable, we necessary to prove odd number 2m+1 can be become a pure 2^n class natural number after it is multiplied by 3 and then added to 1 .Which is make known that we can be prove that Indefinite Equation

$$3\times(2m+1)+1=2^n \quad \cdots\cdots\cdots (1)$$

is tenable.

Equation (1) can be rewritten as

$$6m+3+1=2^n$$

i.e. $$3m+2=2^{n-1}$$

Evident m is even number in above equation .We given m=2D (D>0,positive integer) ,then

$$3D+1=2^{n-2}$$

D is odd number absolutely.

$$D=(2^{n-2}-1)/3 \quad \cdots\cdots\cdots (2)$$

In order to let Equation (2) have positive integer solution,n-2 must be even number. We given n-2=2s (s>0,positive integer) ,then

$$D=[(2^2)^s-1]/3$$

$$D=(4-1)(4^{s-1}+4^{s-2}+\cdots\cdots+4^2+4+1)/3$$

$$D=4^{s-1}+4^{s-2}+\cdots\cdots+4^2+4+1 \quad \cdots\cdots\cdots (3)$$

All corresponding to every S value in Equation (3) have one odd number D:

S=1,2,3,4,5······

D=1,5,21,85,341······

Indefinite Equation (3) is tenable and have infinite many groups of solution. In Indefinite Equation (1) is tenable and have infinite many groups of solution. Infinite many odd numbers D of non-pure 2^n class natural number are exist in the set of positive integers. Odd numbers D can be become a pure 2^n class natural number after multiplied by 3 and then added to1.

So as the calculation is repeated and continued by use of the method described according to "hail"conjecture. Every one optionally of odd number 2m+1 are can be become an odd number D?

So-called odd number is the number in which there is only one or more than one odd prime number divisors 2m+1 (m>0) . "If it is odd number, it is multiplied by 3 and then added to 1." In fact, the purpose of this doing is to change the odd prime number divisors making up a non-pure 2^n class natural number. The new natural number composed by that it is multiplied by 3 and then added to 1 can not be divided with no remainder either by all prime number divisors in the original natural number or by odd prime number 3, so it doesn't contain either odd prime number divisors of the original natural number or odd prime number 3 divisor. For example: $385=5\times7\times11$,$385\times3+1=1156$, $1156=2^2\times17^2$, in which 1156 contains neither odd prime number divisors(5, 7, 11)of the original natural number nor odd prime number 3 divisors. Thus, the non-pure 2^n class natural number

composed of the original odd prime number divisors is excluded to make up a new natural number. As for an optional natural number, according to the calculating method described in "hail" conjecture, no cycle relation exists between other pure 2^n class natural number and all non-pure 2^n class natural numbers, with the exception of pure 2^n class natural number 4—2—1(This cycle relation can not be taken into consideration after the result from the first calculating is 1) This relation is established by the following theorem.

THEOREM: An optional natural number greater than 1, if even number, is divided by 2; if odd number, multiplied by 3 and then added to 1. The result shows no cycle relation exists.

PROOF: When an optional natural number greater than 1 is the pure 2^n class natural number ($n>0$), it is divided by 2 for n times and its quotient decreases at the geometric series whose common ratio is 1/2. The final result is 1. Obviously, no cycle relation exists.

When an optional natural number greater than 1 is the even number $2^n(2m+1)$ ($m>0,n>0$) of non-pure 2^n class natural number , the quotient in the process which it is divided by 2 for n times decreases at the geometric series whose common ratio is 1/2 also. The final result is $2m+1$. Also, no cycle relation exists.

When an optional natural number greater than 1 is the odd number $2m+1$ of non-pure 2^n class natural number, if there is cycle relation in the process of calculating, because $2m+1$ is odd number, but $(2m+1)\times3+1$ is even number, it must satisfy:

$$(2m+1)\times3+1=2^n(2m+1)$$

$$3(2m+1)-2^n(2m+1)=-1$$

$$(3-2^n)(2m+1)=-1$$

in which $2m+1\geqslant3$ obviously.

When $n=1$, then

$$3-2^n=1$$

$$(3-2^n)(2m+1)\geqslant3$$

i.e.

$$(3-2^n)(2m+1)\neq-1$$

When $n>1$, then

$$3-2^n\leqslant-1$$

$$(3-2^n)(2m+1)\leqslant-3$$

id

$$(3-2^n)(2m+1)\neq-1$$

The said hypothesis can't be tenable. No cycle relation exists in $(2m+1)\times3+1$. The proof of the theorem has been completed.

According to the said theorem, one related non-pure 2^n class natural number is excluded from every step calculation of an optional natural number, in compliance with the calculating method described in "hail" conjecture, by which the relation between an optional non-pure 2^n class natural number and other non-pure 2^n class natural number is different; some bear no relation completely, some have less relation and some have more relation, so the non-pure 2^n class natural numbers with no relation can be ignored. However, all non-pure 2^n class natural numbers are excluded from "such repeated calculation". In addition, there are an infinite of odd number D of non-pure 2^n class natural numbers in natural number set, so an optional non-pure 2^n class natural number possibly becomes a odd number D of non-pure 2^n class natural number at any time on condition that odd prime number divisor is changed to make up a

new natural number. For example: 21 is a odd number D of non-pure 2^n class natural number, but it has become a pure 2^n class natural number after one step of calculation, i.e. $21\times3+1=64$. Some non-pure 2^n class natural numbers can become odd number D of non-pure 2^n class natural numbers only for a longer process of calculation. For example: 27 has become odd number D of non-pure 2^n class natural number after 106 steps of calculation. Odd number D has become a pure 2^n class natural number 16 after one step of calculation again. Theoretically, some fully greater non-pure 2^n class natural numbers, even if in the much longer process of calculation, have not become odd number D of non-pure 2^n class natural numbers yet. However, the quantity of non-pure 2^n class natural numbers related to these definite natural numbers in the given interval in the sat of natural number is finite. In the way in which "such repeated calculation" described in "hail" conjecture is continued, that is to say, the calculation which the related non-pure 2^n class natural numbers are excluded is continued, the related non-pure 2^n class natural numbers are excluded one by one. At last, the odd number D of non-pure 2^n class natural number remains only to be the natural number related to the non-pure 2^n class natural number, which was given initially. When odd prime number divisors of the previous non-pure 2^n class natural number is changed again to make up a new natural number, this natural number becomes the odd number D of non-pure 2^n class natural number inevitably. Odd number D has become a pure 2^n class natural number after one step of calculation." Such repeated calculation," an optional non-pure 2^n class natural number which was given initially becomes a pure 2^n class natural number inevitably, then it is divided by 2 for n times, and " the final result is inevitably to be 1," as described in "hail" conjecture. In the case that it is initially to be an optional non-pure 2^n class natural number, "hail" conjecture can be proved in the same way also. "Hail" conjecture has been proved completely.

3 CONCLUSIONS

The essential characteristic of natural number greater than 1 means that this natural number is composed of prime number divisors, which are given. According to the essential characteristic of natural number, natural numbers can be redefined: Natural number is the sum of numbers, which 1 and combined with all prime numbers and two or more than two prime number divisors. According to the difference in the essential characteristic of natural number, natural numbers can be reclassified into two classes: The first class is pure 2^n class natural numbers, i.e. $1(2^0=1)$ and the number which one or more than one prime number divisors are 2; The second class is non-pure 2^n class natural numbers. They are natural numbers 2^n (2m+1) ($m>0,n\geqslant0$),including the number which has only one odd prime number divisor and the number which has two or more than two prime number divisors in which there is one odd prime number divisor at least. By use of the new classification and theorem of natural numbers, the essential law of "hail" conjecture is found, and an optional natural number in "hail" conjecture is chosen, of course, which is either even number or odd number and either pure 2^n class natural number or non-pure 2^n class natural number. "If it is even number, it is divided by 2." In fact, prime number 2 divisors in this natural number are to be reduced, till they are eliminated. If it is pure 2^n class natural number, it is divided by 2 for n times, and the final result is 1. In the case of pure 2^n class natural numbers, "hail" conjecture has been proved. "If it is odd number, it is multiplied by 3 and then added to 1." In fact, the purpose of this doing is to change the odd prime number divisors making up a non-pure

2^n class natural number, and the non-pure 2^n class natural number composed of the original odd prime number divisors is excluded. Through "such repeated calculation," an optional non-pure 2^n class natural number becomes a pure 2^n class natural number inevitably, then it is divided by 2 for n times, and "the final result is 1." In the case of an optional non-pure 2^n class natural number, "hail" conjecture can be proved in the same way also. "Hail" conjecture has been proved completely. It has been generally proved that "hail" conjecture is tenable for all natural numbers.

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后记：我是怎样研究现代数学四大猜想证明的？

天地生出我，大概就是要我去解决一些数学难题的。这可能就是我的人生使命，或者说就是我的历史使命吧？

我估计我也可能落得像费尔马和哥德巴赫一样的下场，尴尬地被别人强迫戴上虚假的数学家的高帽。因此，我赶紧郑重声明：我不是大学数学专业出身的，从严格的传统意义上来说，我是一个数学外行。当然，从来就没有人能够禁止不是数学专业出身的人去研究任何数学问题。如果解决了一个或几个难题的人也可以宽泛地称“家”，也可以看成在那一个或几个问题上的学术权威的话，那么就另当别论了，在数学史上就应该有 1+1=2 的数学家， 1+2=3 的数学家， 2+2=4 的数学家等等数不胜数的数学家。如此，我也可以被忝列为“ 1+1=1 ”等的数学家了。其实，只要解决问题，没有必要这样称“家”。称“家”不是解决问题的办法。当“家”的权威也有解决不了的问题。我仍然认为自己是一个数学外行。我也许正是一个数学外行，才适合去解决上述那些数学难题，因为第一、提出上述猜想的人本身实际上都是数学外行：费尔马是法国的一个律师，哥德巴赫是普鲁士的一个外交官，四色猜想最初是由英国的一个年轻的绘图员法兰西斯·古特里发现的，而冰雹猜想最先是德国的一个大学生考拉兹进行研究的。我是数学外行，就具有类似于作者的思想类型，有类似的思路，有类似的证明方法，就能够不带任何偏见地依照作者的原意去进行研究；第二、我是数学外行，就不会陷入传统数学研究范式，特意去追求那些复杂难懂的间接证明，反而轻视简单明了的直接证明。我的思想没有任何框框，完全自由开放，有利于创造出各种新的证明方法。正是我这个数学外行，把上述现代数学四大猜想都直接证明出来了，它们都一般地成立。别人可能会说我胆大包天，或者骂我是“数学疯子”，那也没有办法。我们只能承认事实，只能服从真理。

我也说不清楚我为什么与数学结下了不解之缘，甚至几次有意想摆脱它也摆脱不了。或许这是出于人的本性。人的本性就是追求知识和真理。我从小就钟情数学，因为它给我荣誉，促我自尊。数学在我每次生活转折的考试中都立了头等功，因为数学问题有明显的确定性，有客观标准，它总是可以获得高分，而且还能够带动物理、化学等自然学科也获得高分。我读书时文科成绩也好，但它们没有数学那种明显的确定性，评卷的人可以带有主观性，一般较难获得满分。我的一生都受到数学的深刻影响。我小学毕业后，首次得益于数学，以全县第二名考上了连平中学初中。1954 年我初中毕业，在广东省原粤北专区的中等学校招生统一考试中，以数学 100 分、化学 98 分、物理 96 分为标致的第二名考上了和平中学高中，获得了两所学校老师的特别厚爱。1957 年我高中毕业时，老师建议我报考大学理科的数学专业。可是那年是我国社会主义建设第一个五年计划的第一年，国家最急需工业建设的科技人才，于是我在国家需要的感召下，没有报考理科的数学专业，而报考了工科的专业。我以优秀的成绩考上了重点大学。进了大学以后，我对数学的兴趣就更大了，除了学好专业课和以微积分为主的高等数学以外，还自学了高等代数、不定方程、数理逻辑和应用数学等。我 1962 年在大学五年制本科毕业后，就很自然地将微积分、优选法和应用数学的方法运用到工程和科学研究上去，并且都取得了很好的成

果。

1978 年初，徐迟发表了散文《哥德巴赫猜想》，大力宣扬陈景润，在我国刮起了一股“哥德巴赫猜想风”。它对于鼓舞知识分子丢掉“臭老九”的黑帽子，振奋精神，解放思想，起了很好的作用。同时它也促进了社会重视知识和知识分子。在那股风中，我头脑清醒，看到了那是形势的需要，是人为的宣传。它本身却是与数学精神相悖的，误导了许多人，造成了严重的后果。在很长的一段时间里，我无意去研究哥德巴赫猜想，但它不可能不在我的思想里留下深刻的印象。由于我当时正在做的工程技术工作和科学研究都需要较多地运用应用数学的知识，脑袋里装满了各种应用数学方法。三月份的一天，我正在市里出差，一个人沉思着走在大街上，头脑里突然产生一个灵感：运用应用数学的方法可以证明哥德巴赫猜想！这个灵感影响了我二十多年的生活，直到现在，给我带来了研究工作的长期艰苦，获得研究成果的短暂惊喜，以及寻求审查、鉴定和出版的苦闷。如果我当时意识到了那个灵感将会把我引入如此崎岖曲折、长满荆棘的数学研究之路，累出了一场大病而险些把小命也丢掉了，甚至取得了成果也很难得到公认，我可能就会把它抛弃掉，现在也就不必在这里啰嗦了。可是我那时年富力强，充满理想，决心要把文化大革命担误的时间夺回来，创造出巨大的科学成就为祖国争光，为人类做出贡献。我紧紧抓住那个灵感，马上跑到新华书店去买了一本华罗庚著的《数论导引》，开始了长期跋涉，钻研起数论来。它的定价是 4 · 60 元，现在看来是多么微少的钱，但在当时却是一个人在城市里半个多月的基本生活费哟！我从那年春末开始对哥德巴赫的证明进行研究，除了上班之外，把全部休息时间和多半的晚上睡眠时间都用了上去，到秋天就写出了论文《哥德巴赫猜想的证明》（现在已改成《哥德巴赫猜想的分析—归纳法证明》），运用应用数学的方法，简单明了地证明了哥德巴赫猜想一般地成立。我把论文呈报给单位，单位组织审查后向省局报告，省局组织审查后又向国家冶金工业部报告。冶金工业部 11 月份通知我到北京去汇报。冶金工业部科技司组织专家对我的论文进行了审查，认为观点明确，方法对路，逻辑性强，论证严密，证明是正确的。审查会后，冶金工业部派人陪我一起去中国科学院数学研究所汇报。一位先生接待了我们。他听了我们的汇报，看了文稿后，一方面说文章本身有道理，但另一方面又说哥德巴赫猜想只能运用理论数学的方法去证明，而且必须按照前人开辟的道路走下去，运用传统的方法，在陈景润“ 1+2 ”的成果基础上去证明，而不能走另外的道路，运用应用数学的方法去证明。当他看到我那时已经有不少白头发时，就劝我不要再去研究了，说那是没有希望解决的难题。他并且说，陈景润只是走狗屎运，前进了一步，但只是前进一步而已。按照那位先生的逻辑，运用计算机去证明任何问题也都是无效的，因为计算机就是运用应用数学的原理制造出来的。我当时对那位先生的奇怪逻辑接受不了，进行了辩论，但没有用，他根本不听。冶金工业部的人也不坚持原则据理而争。我在北京找了著名的大学评理，但那些教授既不说我的论文错，也不说数学研究所的意见对，哼哈推诿了事。我毫无办法，眼睁睁地看着别人把自己科研成果扼杀掉。我的心多么痛苦啊！经过一段时间苦闷，我一方面仍然坚持运用应用数学方法证明哥德巴赫猜想的论文是正确的，另一方面又想到既然运用应用数学的方法能够证明该猜想，那么也一定能够找到证明该猜想的理论数学方法。我振作起来，横下一条心，那就运用理论数学的方法去证明吧！我经过研究以后，认为传统的证明方法是不可取的，必须把它连同前人取得的有关

成果抛开，另外创造新的证明方法。我坚信自己有能力去解决这个问题。可是，不知是哥德巴赫的幽灵对我的如此自信的惩罚，还是有意磨练我的意志，我把十五年的心血和时间投进去了，到 1993 年还是没有取得任何进展。我度过了多少不眠之夜，想出了多少种新方法，写过了多少份草稿，连我自己也记不清楚了，它们都被我一个又一个地否定了。一直到 1994 年夏天，又不知是我长期注入的心血结晶了，还是我的智慧火花爆发了，我的脑子里突然涌现出一个同余分类的新概念。经过整理建构，我创造了一种新的同余分类筛法。通过深入探索，我逐步证明了五条新的定理，从而到 1994 年底再次写出了论文《哥德猜想的证明》(现在已改成《哥德巴赫猜想的同余分类筛法证明》),用理论数学的方法证明了哥德巴赫猜想一般地成立。

我的论文《哥德巴赫猜想的证明》(同上注)，在《河源报》社社长和总编辑的大力支持下， 1995 年 2 月 9 日首次在《河源报》上全文发表了。它在河源市及其周边地区造成了强烈的反响，并且迅速波及全国。我不知道是因为我的论文的发表，还是其它一些类似的或似是而非的新闻报道,有的相关部门好像遭受了一场大火灾。个别领导带头，组织围攻，大泼冷水灭火。他们说什么哥德巴赫猜想的证明，在几十年之内，别说中国，就是全世界，也是无法解决的；说什么有的人宣称哥德巴赫猜想得到了证明，无异于想骑自行车到月球去旅行，谈何容易；说什么谁要想去证明哥德巴赫猜想，就好像想用斧头和锯子去制造人造地球卫星一样不可能；说什么凡是证明哥德巴赫猜想的文章，都必须有两位以上的数论教授的审查和推荐，杂志社才能接受；说什么凡不是用传统的证明方法写出的证明哥德巴赫猜想的文章，都根本不值得看；说什么哥德巴赫猜想的证明，只能寄希望于将来发明了比电子计算机还先进的新手段和新方法之后；等等。其实，比电子计算机还先进得多的手段早就存在了，而且它的历史比电子计算机的历史长得多，可是他们根本就不重视。那是什么？那就是人的头脑。难道人的头脑不是比电子计算机先进几千万倍吗？他们拼命维护传统的旧证明方法，而对于别人创造的新证明方法却装瞎——视而不见，甚至不屑一顾。他们不讲一句有具体内容的实话，尽是笼统地说武断的空话，以权势压人，以名位压人。他们没有针对我的论文，也没有指称我的姓名，我当然只能置之不理。他们上演了一出大喜剧，把学者的风度丢尽，使自己的真面目暴露无遗。我看到他们的文章觉得好笑，但那是一种多么心酸的苦笑啊！不少知情的领导干部、科技界、文艺界、新闻界人士都为我打抱不平，可是没有一点用。

从 1995 年到 2000 年上半年的五年半时间里，我一直在为我的论文《哥德巴赫猜想的证明》(同上注）寻求审查、鉴定和在权威科学杂志上发表而奋斗，可是我到处碰壁，良好的愿望一个接一个落空。后来我才发现：各科学杂志社都有统一的指令，对于有关现代数学三大难题的投稿一律不接收，全部进行封杀。这种学术风气对我当然是一个打击，但毕竟还是个人小事。它的最大危害是会影响我国社会主义现代化建设，阻碍科学技术的发展。我真不理解我们的国家在改革开放的今天，怎么还会容许那么恶劣的学术风气存在的。我回想起 1978 年冬数学研究所的人所说的话，把它们与目前那些人所说的话进行对比，时间相隔二十多年，它们还是同一个腔调，也是出于同一个目的——保持学术垄断地位。我对哥德巴赫猜想的研究，体会最深的是：证明一个数学猜想是极其艰苦的事。我研究哥德巴赫猜想就是长期自找苦吃。而《哥德巴赫猜想的证明》(同上注）一文的遭遇使我最痛心的是：一个重大的科学研究成果要发表出去多么艰难啊！谁要去研究数学猜想，就不可避免地

得罪数学权威，就要不怕危险去进行突围的拼搏。

我虽然对其它数学难题有不少了解，而且还对有的数学难题进行过深入的研究，但我害怕了，再加上自己的年纪也超过六十岁了，就不想再对其它数学猜想进行研究了。可是，在 2000 年 7 月份的一个夜晚，我躺在床上，我的脑子里又突然产生一个灵感：四色猜想可以用数理逻辑的方法简单明了地证明出来。我担心这个灵感又会像哥德巴赫猜想可以证明的灵感那样，给我带来那么多的痛苦，因而极力廻避它，有意打断有关的思路。但是，好像命中注定我要去打破别人所说的该猜想的证明是“不能单独用手工来完成”的神话，非要我把四色猜想证明出来不可似的，四色猜想总是廻避不了，思路越受阻挡，思潮就越汹涌澎湃，不把它写出来就经常睡不着觉。这大概又是自己作为一个知识分子的科学使命感吧？这样坚持了约两个月，我没有办法，只好下决心进一步研究它并把它写出来，以便了却一桩心事，至于它能不能发表，别人会不会公认它，什么都不管吧，反正谋事在人，成事在天，由它去吧！我曾经设想，如果我没有数学知识，不是就没有这么多苦恼了嘛？我既然有一定的数学知识，那就只能忍受生活中相应的痛苦了。我的命好苦啊，进入老年了还不得安宁！我又集中精力进行研究与写作，用了八个月的时间写出了论文《四色猜想的数理逻辑法的直接证明》。在这篇论文中，我首先创造了一种新的相邻区域段的着色方法，把相邻区域段划分为四大类十二类二十一个类型，然后通过数理逻辑的穷举法论证，直接证明了四色猜想完全成立。

我松了一口气，下决心以后再也不干数学猜想的研究工作了。我迅速将我的注意力转移到伴随我走过了大半人生道路的缪斯女神身上，希望用文学艺术的力量把数学猜想从脑海里推出去。我大约过了一年艺术人生的美好生活。到了 2002 年 6 月份，我一不小心，“冰雹猜想”就再一次从我的脑子里冒了出来。到了 22 日晚，这个猜想的证明思路在我的脑子里豁然开朗。它是那样明晰，那么简单，那么美！它就是一个真理摆在我的面前。王蒙在评论“最高的诗是数学”这一句话时曾经指出：“最高的数学和最高的诗一样，都充满了想象，充满了智慧，充满了创造，充满了章法，充满了和谐也充满了挑战……又都充满了灵感，充满了激情，充满了人类的精神力量”。我说：最高的数学就是最高的诗。我非把这些象征着真理又充满着诗意的美好文字写下来不可。我不得不又研究起冰雹猜想来，奋笔疾书，日以继夜，仅用了两个月的时间就把论文《冰雹猜想的证明》写出来了。在这篇论文中，我揭示了大于 1 的自然数的本质特征，重新定义了自然数，对自然数进行了新的分类，证明了一条新的定理，找出了冰雹猜想的本质规律，一般地证明了这个数学猜想对于所有自然数都成立。

冰雹猜想的证明这件事好像告诉我：一个人的使命是躲避不掉的。我这样想，虽然不想再去证明什么数学猜想了，但也不敢说不去做什么了，避免“说话不算数”的罪过。果不其然，我还没有过上半年轻松日子，费尔马猜想便乘虚钻入了我的脑子里。我从 1980 年到 1998 年期间曾经对费尔马猜想进行过深入的研究，并且取得了肯定的中间成果，但后来中断了，四五年来没有集中精力去研究它。说也奇怪，2003 年 2 月 4 日晚，一条新定理的灵感突然出现在我的脑子里，运用它就可以简单明了地直接证明费尔马猜想了。真是“踏破铁鞋无觅处，得来全不费功夫”！我深刻体会到：所谓灵感就是长期集中思想艰苦奋斗的心血，在心情轻松状态下的自然结晶；99%的心血+1%新方法的灵感=成功。我只用了四个月就写出了《费尔马猜想

的初等数学方法的直接证明》的论文。它在前人特殊证明成果的基础上，证明了两条新定理，运用初等数学的方法，简单明了地直接证明了费尔马猜想一般地成立。

随着二十一世纪的到来，我好像突然变得年轻了，精力充沛，思维敏捷，开头三年每年写出了一篇证明数学难题的论文。现代数学的三大难题都被我证明出来了，中间还加上了一个冰雹猜想的证明，我的科学使命应该说完成了吧？数学难题还有很多，但我希望今后我的头脑里不要再出现什么解决纯数学难题的灵感了。如果灵感硬是还要来，那就至多来一两个与人类生活直接有关的重大科学技术灵感吧，我就拼老命再来一两个回合。

我们知道，数学的特点就是具有明显的确定性。数学的每一个正确命题和论证都是一个真理。这正如伽利略曾经说的:“数学是上帝的语言”。不过，真理又往往必须冲破重重障碍才能得到公认。费尔马猜想、四色猜想、哥德巴赫猜想和冰雹猜想，经过我的论证，证明它们都是正确的。它们都是真理。它们今后应该分别改称为同名的定理，即费尔马大定理，四色定理，哥德巴赫定理和冰雹定理。读者从本书的五篇论文中可以看出，我的证明是完全正确的，当然也是真理。我坚信：真理必然胜利。

作者 2005 年 10 月

Postscript: How Do I Study

The Proof Of Four Great Conjectures In Modern Mathematics?

Perhaps I was born to solve some difficult problems in mathematics. This is possibly my mission of life or my historical mission.

I guess that my fate maybe will be the same as Fermat's and Goldbach's who were worn by illusive crowns of mathematics embarrassingly and unwillingly. Therefore I have to hastily declare that I am not of mathematics major, and to be strict I am a layman of mathematics. Of course there have never been any persons who can forbid non-mathematics-majors to do research on mathematics. If only those who can solve one or a few difficult problems can be called specialists or authorities in academic fields, then it is another thing. In the history of mathematics there should have been mathematicians of 1+1=2, of 1+2=3 and of 2+2=4 and so on. If so I might be added as a mathematicians of "1+1=1". As a matter of fact, there is no need to be called specialists or not so long as the problems can be solved. I still consider myself a layman of mathematics. Perhaps just for that I am a layman of mathematics I am suitable to solve some of the above-mentioned difficult problems in mathematics. First of all because the above mentioned conjectures of mathematics were put forward by laymen of mathematics. Mr. Fermat used to be a lawyer in France. Mr. Goldbach used to be a diplomat in Prussia. The Four-colour conjecture was first discovered by Mr. Francis Guterry, a young draftsman of Great Britain. And the Hail Conjecture was studied for the first time by Mr. Kaulatz, a German college student. I am a layman of mathematics, That is why I have similar ideas, similar ways of thinking and similar methods of proving as they had and I can study their conjectures according to their original intention without any prejudice. Secondly, because I am a layman of mathematics, I cannot be involved in the traditional modes of mathematics research to seek those complex and hard-understandable indirect proving instead of ignoring the understandable direct proving. My thinking is free and open up without any frames, which is more suitable to create different kinds of new proving methods. I am a layman of mathematics who proved the above mentioned great four conjectures in modern mathematics and the proving is generally established true. Maybe some people think I am so daring, abuse me a mathematics madman. But that is a fact and truth, which everybody can to admit and obey only.

I cannot explain why I have been destined to mathematics so closely. I was going to get rid of it for several times but I could not do so. Perhaps this is start from human natural instincts. Human natural instincts are seek knowledge and truth. I began to enjoy mathematics very much since my childhood and it brought me honor and self-respect. In many examinations of mine mathematics had helped me a lot. I could always get high

grades since mathematics has clear definiteness and has objective criteria. And it also helped me to get high grades in physics and chemistry. I was good at liberal arts as well, but they do not have clear definiteness like mathematics. While judging the papers the teachers usually have some subjective factors, so it is hard for the students to get full marks. I have been influenced by mathematics deeply for my whole life. After I graduated from my elementary school I was admitted by Lianping Junior High School because of high grades of mathematics. And in 1954 when I graduated from junior high school I took part in the unified examinations of Northern Guangdong Prefecture and was admitted in the second place because of high grades (mathematics 100 points, chemistry 98 points, physics 96 points) by the No 1 Senior High School of Heping County. In these two schools, I had been loved by the teachers. In 1957 when I graduated from the senior high school, my teachers suggested me that I should apply for the major of mathematics in university. However it was the beginning of the first year of the first Five-year Plan for Socialist Construction of China and the country needed scientific and technical talented persons for the social construction anxiously. Therefore I applied for the technical majors of industrial universities rather than for the mathematics major of scientific universities. As a result I was admitted by a key industrial university because of excellent grades. While studying in the university I was more interested in mathematics. Besides major courses and the advanced mathematics, calculus, I also studied advanced algebra, indefinite equation, mathematic logic and applied mathematics by myself. After five years' studying in the university I graduated from it in 1962. While working as an engineer afterwards I made full use of my mathematics knowledge such as calculus, the best option approach and applied mathematics in the engineering work and scientific research. Due to mathematics I achieved excellent results in my work.

In 1978, a Chinese famous writer, Xuchi published an essay entitled "The Goldbach Conjecture" in the newspaper, which blazoned forth mathematician Mr. Chen Jingren and started a gust of wind of Goldbach Conjecture in China. This event played a good role in encouraging the intellectuals known as "Stinking No.Nine" at that time to emancipate their thoughts and revivify themselves. And it also promoted the whole society to attach more importance to the intellectuals. In those days of the wind I was clear-headed knowing that it was a contrived propaganda to meet the needs of the political situation which was opposite to the mathematics spirit and led the people in a wrong way having brought very serious results. At that time I had no intention to study Goldbach Conjecture but it was impossible for me to be left a deep impression in my mind. Because I was busy doing my engineering and research work at that time, my mind was full of different kinds of applied mathematics methods. One day in March when I went on business to the city walking and thinking in the street, an inspiration came to my mind suddenly: Golsbach Conjecture can be proved by means of applied mathematics. It was this inspiration that influenced my later life for more than 20 years up to now, which have brought me a long time hardship in my research work, a short time of surprise and joy of my research achievement and the agony in seeking checkup, appraisal and publishing. If I had realized

that the inspiration would lead me to such a road of mathematics research full of hardship that I almost died because of a serious disease and had known that the research results could not be recognized, I would have dropped my research work and would have had nothing to talk about here. However, at that time I was in my golden age full of energy and ideals being determined to grab back the lost time caused by the Great Cultural Revolution to create great scientific achievement within my ability for the honor of our country and to make my contributions to the mankind. Following that inspiration in my mind I began to do my research form reading "The Introduction to Theory of Number" by the famous Chinese mathematician Luogeng Hua. I started my research of proving Goldbach Conjecture form the late spring of that year working at it day and night with all my efforts. And in the fall of the same year I wrote out the treatise "The proof of Goldbach Conjecture" (now it has been changed as "The Proof Of Goldbach Conjecture By Analysis-Induction Method") which proved its establishment simply and clearly by means of applied mathematics. Then I submitted the treatise to the authorities of my working unit and then it was submitted to the provincial even the central administration authorities. In November I was called to Beijing to report my research achievement in the Ministry of Metallurgy. The specialists of the Department of Science and Technology of the ministry checked up the treatise considering that the points of view are clear, the methods used are right, the logic is reasonable and the proving is serious and correct. After the check-up meeting in the ministry I went to the Mathematics Research Institute of the Science Academy of China to report my treatise. A Mr. accepted me. After he heard my report and read the treatise, he said that treatise itself is reasonable, but the Goldbach Conjecture can be proved only by means of the methods of theoretical mathematics following the way opened up by the forerunners with traditional methods and on the basis of Chen Jingren's 1+2 achievement, and it cannot proved by applied mathematics. Seeing that my hair has become gray be persuaded me to stop the research saying that there is no hope to solve this hard problem. And he added that Mr. Chen Jingren was luck to move a step forward, only one step. According to this Mr. s logic, it is also ineffective to prove any problems on computers because computers are deigned and made on the basis of mathematics principles. At that time I could not accept his strange logic and argued with him, but it was no use since he didn't listen to any different ideas. Then I went to some famous universities to visit their professors who didn't say no to my treatise and also didn't say yes to the ideas of the Mathematics Institute. Thus my treatise was denied and I could do nothing at all. My heart was bitterly painful for that. After a period of pang and distress I re-collected myself and continued to do my research. Believing the correctness of proving Goldbach Conjecture with the methods of applied mathematics I thought that since it can be proved by methods of applied mathematics it must be possible to find the methods of theoretical mathematics to prove it. I was determined to use the methods of theoretical methods to prove it. Having done some research on this I found that the traditional methods of proving is not desirable and acceptable and a new method of proving must be created to get rid of the achievements of predecessors in this aspect. And

I was sure that I could solve this problem. However, perhaps because the Goldbach's ghost was punishing my self-confidence or testing my will power on purpose I didn't get any headway until 1993 having put my heart, energy and time of 15 years into the research. I don't know how many wakeful nights I had experienced, how many new methods I had tried and how many manuscripts I had written which I denied one by one.

It was in summer of 1994 when a new concept of congruence classification came to my mind. I don't know whether it is the reward for my long-term hardworking in my research or a sudden burst of my wisdom spark through neatening and establishing structure I created a new Sieve method of congruence classification. And after further exploring I gradually proved five new theorems. As a result I wrote out the treatise "The Proof of Goldbach Conjecture" (now it has bee changed as "The proof of Goldbach conjecture by sieve method of congruence classification") by the end of 1994, which proved its general establishment.

This whole treatise was published in Heyuan Daily September second, 1995 for the first time with the support of the head and editorial. Its publishing caused a great sensation in Heyuan city and surrounding areas. But at the same time it also caused a series of opposition to the treatise. Some people said that it is impossible for China even the world to prove the Goldbach Conjecture within decades of years to come. Some people said that the claim of proving Goldbach Conjecture is similar to traveling to the moon by bicycles or similar to making an artificial planet of the earth with axes and saws. Other people said that any treatise proving Golsbach Conjecture can be accepted by the editorials only after the check-up and recommendation of two professors of number theory and any treatise which was not written with traditional methods of proving is not worth reading. They even said that only after the new means and methods which are much more advanced than computers appeared can the Goldbach Conjecture be proved. As a matter of fact, the means which are much more advanced than computers have long been existing and much longer than the existence of computers, but they are ignored at all. What are they? They are the brains of human beings. Aren't the brains of human beings thousands times more advanced than computers? Those people protect the old traditional proving methods with every effort and completely ignore even not have a glimpse at the new proving methods other people created. They did not say any single word of specific content just saying empty words of general and dogmatical ideas to suppress people by power and authority. They did not refer to my treatise and my name, therefore I did not argue with them keeping silence. They played a big comedy completely losing their academic style and their real face. Having read their critical papers I could not help laughing, but it is a bitter and miserable laugh for me. Quite a few leading cadres, engineers, writers and reporters who know me and my research all thought it is an unfair treatment but they could do nothing as well.

From 1995 to 2000, for five years I had been making all efforts to seek and ask for the check-up, appraisal and publishing of my treatise "The proof of Goldbach Conjecture" (same above annotation) in an authoritative academic journal but I met all

kinds of obstacles everywhere and my fond hopes were dashed to the ground overturn. And finally I found that there is an order for all academic journals: all treatises on the three modern mathematics conjectures are not to be accepted and not allowed to publish. Such an approach to academic research is a small matter to me but it is a very harmful attitude to the socialist construction of our country blocking the development of science and technology. I really do not understand why such an abominable academic attitude can exist while the country is carrying out the policies of reform and opening up. I remember that in 1978 what the persons of the National Mathematics Institute said to me is almost the same as what some people say now after 20 years later. Their purpose is the same—to keep the academic monopoly.

From my research on the Goldbach Conjecture, the most impressive experience I got is that to prove one mathematics conjecture is full of hardship. What I did in the research of the Goldbach Conjecture is just taking troubles by myself for a long time. And most miserable thing is that my research treatise has been suffering from refusal everywhere. How hard it is to publish the result of a great scientific research! Whoever wants to do the research on mathematics conjectures is doomed to offend the mathematics authorities and to take numerous troubles and to fight.

Although I have a wide knowledge of mathematics problems and have done some deep research on some of them, I become afraid doing it now. Besides, I am already over 60 years old so I do not want to do it any more. However, one night in July, 2000, while I was lying in bed an inspiration suddenly came to my mind: the Four-colour Conjecture can be simply proved with the method of mathematical logic. But I was worried about the same miserable result as I experienced before so I tried hard to escape from it and even intended to drop it. However, it seemed to me that I am destined to break the myth that "this conjecture can not be proved only by single hand", and it needs me to prove it. Thus I could not avoid it. The greater the obstacle is, the stronger my determination is. I could not sleep at night without thinking about it and doing it. Perhaps this is the scientific mission of me as an intellectual. After about 2 months had passed like this, I made my mind to do this research and to write out the treatise to settle my worry. I do not mind whether it can be published or not or whether the others accept it or not. I minded nothing at that time. Action depends on people and success depends on gods. Let it be as it will be! I sometimes thought that if I had not had mathematics knowledge, I would not have been so miserable. However now that I do have certain mathematics knowledge, I have to stand the sufferings in my life. How miserable my life is! I can not live a calm life even during my late years. After 8 months of concentrating hard work I finished my treatise "The Direct Proof of Four-Colour Conjecture by Method of Mathematical Logic". In this treatise I created a new coloring method among the neighboring areas and then divided neighboring areas into 4 groups, 12 kids and 21 types. In the end the Four-Colour Conjecture was proved to be completely true by the close seeking method of mathematical logic

Thus I felt relieved at last making my mind never to do research on mathematics

conjectures any more. Soon I turned my attention to the goddess Muse who accompanied me for the most part of my life hoping to get rid of mathematics conjectures from my mind by the power of literature and art. Therefore I had been leading a wonderful life with art for about one year. However, in June 2002, carelessly I fell into the thought about the Hail Conjecture once again. Up to the night of June 22nd, a clear approach to prove this conjecture came to my mind, which was so clear, so simple and so beautiful! It was a truth appearing in front of me. The writer Wang Meng wrote in his essay once like this: "The most advanced poem is mathematics. The most advanced mathematics is similar to the most advanced poetry being full of imagination, wit, creation, organization, harmony, challenge, inspiration, passion and human spiritual power." And I say that the most advanced mathematics is the most advanced poetry. I must write out the wonderful words that represent the truth and are full of poetry meaning. Thus I had to begin my research on the Hail Conjecture studying and writing hard day after night. And I finished my treatise "The Proof of 'Hail' Conjecture" only 2 months later. In this treatise I revealed the essential features of the natural numbers larger than one and gave a new definition to the natural numbers, classifying the natural numbers newly and proving a new theorem. As a result the essential laws of the Hail Conjecture had been found out, which generally proved that this conjecture is true to all natural numbers.

It seems that from the proof of Hail Conjecture, I realized that a person's mission is not to be avoided. I thought that I would not try to prove any mathematics conjectures any more, but I could not promise to do nothing in future. However, it was not after a half year that the Fermat Conjecture came to my mind again. I had done some deep research to it from 1980 to 1998 and had gained some positive intermediate achievement. But the work stopped later for about 5 years. It was strange that at night February the 2nd 2003 an inspiration of a new theorem came to my mind, with which the Fermat Conjecture can be simply and directly proved. From my experience I got a deep understanding that an inspiration is a natural crystal of a long-term concentrated thinking about a problem through hard working appeared in a relaxed spiritual situation. I finished my treatise "The Direct Proof of Fermat Conjecture by Method of Elementary Mathematics" only in 4 months, which proved two new theorems on the basis of predecessors' proving achievements and simply proved the general establishment of Fermat Conjecture by method of elementary mathematics.

As the time entered the 21st century, it seems to me that I become younger than before being full of energy with sharper thinking and being able to write out a treatise on hard mathematics problem every year over the past three years. Thus I have proved the three hard problems in modern mathematics and the Hail Conjecture as well. I thought that perhaps I have finished my scientific mission. Of course, there are still a lot of hard mathematics problems to be resolved but I hope that there would not be any inspiration appearing in my mind any more. If it does come to me again, I hope only the inspiration about one or two problems directly related to important science and technology for human life would come at most and I would fight for another one or two rounds with my

human life would come at most and I would fight for another one or two rounds with my remaining energy.

We known that the characteristics of mathematics is its definiteness. Every one correct proposition and demonstration of mathematics is a truth. This as Galileo had said: "mathematics is God' s language". And the truth usually can be accepted by public only after it breaks all kinds of obstacles. I have proved the Fenmat Conjecture, the Four-colour conjecture, the Goldbach Conjecture and the Hail Conjecture to be true. They are all truths. Therefore these conjectured should be renamed as theorems, that is the Fermat Last Theorem, the Four-colour Theorem, the Goldbach Theorem and the Hail Theorem. From the five treatises of mine in this book the readers can see that my proving is completely correct, which of course is a truth too. I firmly believe that truth is doomed to win in the end.

The author

October,2005